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OF THE

ROYAL ASTRONOMICAL SOCIETY.

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No. 5.

SIR DAVID GILL, K.C.B., F.R.S., PRESIDENT, in the Chair.

John Edward Austin, West Court, Detling, near Maidstone ;
Rear-Admiral H. E. Purey Cust., R.N., Hydrographer of the
Navy, Admiralty, Whitehall, S.W. ;

Evelyn G. Hogg, M.A., Christ's College, Christchurch, New
Zealand ; and

Albert Henry Stuart, B.Sc. Lond., 52 Greatheed Road, Leam-
ington Spa,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows
of the Society, the names of the proposers from personal knowledge
being appended :—

Lionel Barton Booth, Optician, Aberdour, Pinner Hill Road,
Pinner (proposed by F. J. M. Stratton) ;

Major Gerald Edward Galbraith, R.A., c/o Messrs. Cox & Co.,
16 Charing Cross, S.W. (proposed by E. E. Markwick) ;

Thomas King, Government Astronomical Observer for New
Zealand, 58 Ellice Street, Wellington, New Zealand (pro-
posed by James McKerrow) ;

E. H. V. Melville, P.O. Box 719, Johannesburg, South Africa
(proposed by E. T. A. Innes) ;

John Rees, Chartered Secretary and Fellow of the Corporation

of Accountants, Snowdonia, Miskin Street, Cardiff (proposed by W. H. Rees); and
 Edgar C. Willis, Southwell Lodge, Ipswich Road, Norwich
 (proposed by A. C. Allen).

One hundred and twenty-five presents were announced as having been received since the last meeting, including, amongst others:—

Cambridge Observatory, Measures of Double Stars made under the direction of Professor Challis, 1839–1844, presented by the Observatory; Sir G. H. Darwin, Scientific Papers, vol. iii., presented by the Author; Oxford University Observatory, Astrographic Catalogue, vol. vi., presented by the Observatory; Paris Observatory, Atlas photographique de la lune, fasc. 11, presented by the Observatory; G. V. Schiaparelli, Misura di Stelle doppie, 2 vols., presented by the Author; Spitzbergen, Missions scientifiques pour la Mesure d'un Arc de Méridien, Mission Russe, tome 1, sect. ii., presented by the Poulkovo Observatory.

Astrographic Chart: 33 charts from the Algiers, Paris, and Toulouse Observatories, presented by the French Minister of Public Instruction; Percival Lowell, photographs of Mars, Jupiter, and Saturn, presented by Sir W. Huggins.

Portrait of Warren De la Rue, presented by Mr. Ernest De la Rue.

The systematic motions of the Stars shown in the cross proper motions of the Bradley Stars. By Professor F. W. Dyson, M.A., F.R.S.

In No. 9 of the Publications of the Groningen Laboratory, Professor Kapteyn gives the resolved parts of the proper motions of the Bradley Stars in the direction of and perpendicular to that of the solar apex. In the diagram, if P is the north pole and A the solar apex, taken to be at R.A. $18^h 12^m$, Dec. $29^\circ 5'$, then the components of the proper motion of each star are given, v in the direction SA, and τ perpendicular to it. The component v is made up of the resolved part of the peculiar motion of the star and of the solar motion, but the component τ is simply the resolved part of the peculiar motion of the star. If the stars are arranged according to the azimuthal angle PAS, we shall obtain a series of components of proper motions whose *directions of motion* are in different azimuths. Separating the positive and negative values of τ , the cross proper motions may be arranged in groups for each 10° (or some convenient unit) of azimuth to which they are moving. If the peculiar motions of the stars showed no preference for any direction or directions, these cross proper motions would show the same distribution for each group. But if there is any

and those of unknown type. Only those of known type have been considered here. So that the *data* may be fully understood, the actual cross motions are given in the following tables. When T is positive, the angle A is formed by adding 90° to PAS , and when negative by subtracting 90° . The different columns give the cross motions for values of A $0^\circ-10^\circ$, $10^\circ-20^\circ$, etc., arranged in order of magnitude. For more easy comparison of the proper motions in diametrically opposite directions, the groups 180° away are printed side by side.

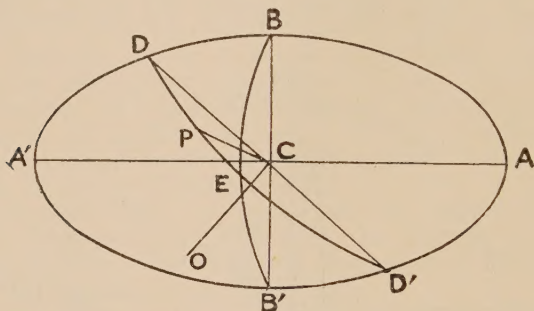


DIAGRAM 2.

Referring again to diagram 1, the figures given in the table in the column headed 5° are the cross motions which are directed to within $\pm 5^\circ$ of the point marked 5° in the diagram; the figures in the column headed 15° are the cross motions directed to within $\pm 5^\circ$ of the point marked 15° , and so on for the whole of the great circle whose pole is A . The annual cross motions in each column are arranged in order of magnitude, the unit being $''\cdot001$, so that 330 denotes an annual proper motion of $0''\cdot330$.

TABLE I.

Cross proper motions of Stars of Type II. in different Azimuths.

Unit="001.

A.	5°.	185°.	15°.	195°.	25°.	205°.	35°.	215°.	45°.	225°.	55°.	235°.
1	330	314	556	1276	3142	4001	151	477	558	472	405	285
2	281	289	352	639	451	703	150	377	299	408	184	134
3	236	242	187	545	255	656	124	361	221	195	177	120
4	157	156	157	521	217	523	112	230	174	107	113	82
5	145	130	150	515	213	330	102	172	148	77	104	62
6	144	115	121	461	146	328	96	169	120	63	96	54
7	115	114	101	435	134	323	93	168	115	57	89	50
8	112	98	90	342	112	317	92	122	109	48	88	48
9	111	87	84	292	102	257	83	95	68	37	69	47
10	108	82	82	173	101	190	68	94	58	37	45	37
11	100	76	77	166	98	189	58	84	50	32	44	63
12	92	53	69	154	92	166	29	70	45	32	43	32
13	90	51	68	110	86	156	29	59	40	28	39	32
14	77	33	65	87	74	145	25	54	32	28	34	28
15	73	29	61	83	71	119	24	39	32	28	27	22
16	70	26	58	74	68	102	18	36	26	27	26	19
17	64	20	55	71	66	91	17	31	21	22	24	19
18	55	17	51	68	64	86	13	31	7	21	20	19
19	50	14	47	66	61	70	12	27	5	18	19	18
20	49	12	40	55	57	66	11	24	3	17	14	16
21	48	8	40	52	51	57	10	23	2	14	12	16
22	48	6	38	52	46	50	9	18	1	11	10	15
23	44	2	33	49	45	49	9	15		10	10	15
24	39	2	31	41	44	46	7	14		9	10	15
25	30	1	30	39	39	46	7	9		9	10	10
26	23		26	38	33	43	7	6		1	9	8
27	22		19	36	31	43	7	6			6	7
28	20		16	35	30	40	4	5			5	4
29	17		15	34	26	40	3	3			3	4
30	15		14	32	24	36	2	1			2	3
31	15		12	31	15	35	1				2	3
32	12		11	30, 0	14	35					0	2
33	12		5	28, 0	13	27						2
34	8		5	27, 1	12	26						2
35	7		2	26, 1	11	24						0
36	7		0	25, 2	10	23						
37	4			20, 2	9	22						
38	4			15, 7	9	14						
39	2			14, 7	8	12						
40	2			14, 7	6	12, 0						
41				13, 9	6, 1	9, 5						
42				12, 10	2, 2	8, 6						

TABLE I.—*continued.*

A.	125°.	305°.	135°.	315°.	145°.	325°.	155°.	335°.	165°.	345°.	175°.	355°.
1	217	270	418	974	344	680	374	1111	999	703	1412	390
2	205	355	289	556	314	512	540	314	655	531	605	316
3	143	186	260	224	312	258	203	187	275	403	552	288
4	134	180	90	126	247	209	191	171	233	314	540	281
5	125	145	88	125	232	194	143	103	224	246	406	222
6	115	129	46	110	214	185	132	76	219	191	331	180
7	86	97	22	92	101	127	98	74	184	184	315	172
8	77	75	19	86	70	122	96	67	161	159	307	168
9	67	74	18	85	65	112	91	67	147	141	237	137
10	60	53	15	79	53	80	89	61	144	139	215	111
11	54	51	11	78	52	64	81	58	136	135	214	105
12	53	44	10	68	38	59	74	58	134	133	208	80
13	38	38	9	64	30	53	74	49	125	129	187	80
14	24	37	8	64	21	51	52	47	77	125	181	80
15	22	36	8	54	21	51	48	44	72	117	159	79
16	21	25	6	52	20	48	45	42	70	93	140	73
17	15	19	6	49	15	46	42	42	70	86	112	51
18	13	17	4	43	13	45	31	40	59	80	106	48
19	10	15	3	41	12	45	29	36	54	72	81	46
20	8	12	2	34	10	37	29	32	51	36	58	38
21	3	11		25	9	34	25	31	36	36	58	37
22	0	9		25	9	29	23	25	33	30	49	36
23		9		23	8	20	18	24	31	28	48	30
24		8		21	7	19	16	19	31	26	48	25
25		6		21	3	12	10	19	28	21	48	22
26		4		17		12	10	15	28	21	46	21
27		3		16		10	9	12	27	17	45	20
28		2		13		8	6	10	24	15	44	18
29				11		7	3	10	23	14	42	14
30				10		5		8	22	14	42	14
31				8		3		6	21	13	34	13
32				7		2		3	21	13	29	11
33				3		2		1	18	13	29	6
34				3		2		0	17	8	28	4
35				1		1			17	8	27	2
36						1			16	7	26	1
37									11	4	18	1
38									9		18, 4	1
39									7		18, 6	
40									6		16, 6	
41									6, 1		12, 7	
42									6, 2		10, 9	

TABLE II.—continued.

[illegible]

TABLE II.—*continued.*

A.	125°.	305°.	135°.	315°.	145°.	325°.	155°.	335°.	165°.	345°.	175°.	355°.
1	68	103	160	651	330	153	196	210	122	176	190	180
2	65	74	64	180	128	153	78	157	61	160	126	105
3	62	70	61	96	54	129	42	99	58	155	124	84
4	54	65	59	73	44	89	40	88	43	91	81	62
5	45	57	49	72	32	64	37	76	34	84	73	60
6	42	40	47	49	26	63	34	73	30	83	63	59
7	37	34	47	43	26	61	30	70	30	80	59	56
8	36	31	40	40	24	56	30	69	23	51	54	56
9	36	31	39	31	22	50	28	68	19	45	44	55
10	33	30	38	30	14	44	28	60	16	44	43	53
11	32	29	30	29	11	44	22	60	12	43	43	51
12	28	25	30	29	11	36	22	50	11	40	36	44
13	25	22	27	28	9	34	21	47	8	40	31	42
14	21	21	25	25	9	28	19	40	8	39	30	40
15	20	20	23	24	8	28	19	39	5	38	26	38
16	18	18	14	23	5	27	19	37	3	33	22	36
17	18	16	14	22	3	23	18	28	3	31	21	34
18	13	16	13	21	3	23	17	26	3	30	20	31
19	9	10	10	21	1	21	14	26	3	30	16	26
20	9	9	10	21		20	12	25	2	29	15	26
21	8	8	8	20		14	11	20	2	29	14	26
22	6	8	6	18		14	9	17	2	27	11	25
23	6	7	6	13		12	9	16	1	26	9	25
24	6	5	6	12		11	9	16	1	26	7	20
25	6	5	6	11		11	7	12	1	24	6	15
26	5	1	4	8		11	6	11	1	21	5	13
27	5	0	0	4		9	5	10		20	4	12
28	2			3		9	4	9		20	3	11
29	2			3		8	3	6		17	3	8
30	1			3		8	1	6		15	2	6
31	1		1			7	0	6		11	1	4
32	0		1			6		5		11		4
33			1			5		4		9		4
34						5		4		9		4
35						4		2		9		3
36								2		9		1
37								1		8		
38								1		7		
39								0		5		
40								0		3		
41										3		
42										2		
										2		

In the above tables the number of stars in any group depends entirely on the distribution of the stars in the sky, and is independent of the proper motions. A cursory examination of the lists

shows, especially for type II., a systematic variation of the magnitudes of the proper motions with the azimuth. In Table III. the middle term of each list and the mean of approximately one-fifth of the numbers grouped about the median are given. Thus 48.5, the first figure of column 3, is the mean of Nos. 20 and 21 of Table I, column 1, and 49.6 is the mean of Nos. 17-24 of the same column.

TABLE III. Unit="001.

Limits of Azimuth.	Type II.			Type I.		
	No. of Stars.	Median P. M.	Mean of Middle Fifth.	No. of Stars.	Median P. M.	Mean of Middle Fifth.
0-10	40	48.5	49.6	23	31	27.4
10-20	36	49	48.7	30	23.5	22.5
20-30	44	45.5	47.0	35	14	14.7
30-40	31	18	19.7	37	19.5	20.1
40-50	22	47.5	48.2	34	8	8.3
50-60	32	25	25.0	30	6.5	6.8
60-70	23	11	12.6	48	10	9.7
70-80	20	17	16.8	48	19	18.6
80-90	23	21	23.8	31	18	16.7
90-100	17	20	20.7	16	14.5	14.5
100-110	27	22	22.2	18	10	11.8
110-120	22	23.5	23.3	17	18	18.7
120-130	22	53.5	51.3	32	18	16.5
130-140	20	13	13.5	27	25	23.8
140-150	25	30	32.4	19	14	15.7
150-160	29	48	52.2	31	19	18.1
160-170	44	32	36.5	26	8	7.8
170-180	47	48	49.0	31	22	23.7
180-190	25	51	48.4	25	15	17.0
190-200	53	36	37.9	31	21	21.6
200-210	45	49	52.2	29	19	19.0
210-220	30	37.5	41.7	41	11	11.3
220-230	26	28	29.2	64	12.5	12.5
230-240	35	19	18.4	38	9.5	10.8
240-250	19	19	19.3	34	8	8.5
250-260	23	23	22.0	37	13	12.9
260-270	26	29.5	31.2	20	11	12.5
270-280	25	20	21.6	23	14	14.2
280-290	28	29.5	31.0	26	15.5	15.3
290-300	27	23	26.4	31	15	13.9
300-310	28	36.5	33.2	27	21	21.2
310-320	35	43	42.6	33	22.5	22.4
320-330	36	45	42.5	35	23	22.3
330-340	34	41	39.3	40	22.5	21.7
340-350	37	72	61.9	43	27	26.9
350-360	38	42	44.9	36	26	29.1

The proper motions are, roughly speaking, largest about 0° and 180° , and smallest about 90° and 270° . The above table shows a

good deal of accidental variation, and therefore larger groups must be taken. The agreement of the *median* and the mean of the middle fifth shows that no appreciably better accordance can be obtained by treating the figures in other ways so long as these small groups are retained.

Taking limits of azimuth of 30° the following values are obtained:—

TABLE IV. Unit="001.

Limits of Azimuth.	Type II.		Type I.	
	No. of Stars.	Median Proper Motion.	No. of Stars.	Median Proper Motion
$0^\circ-30^\circ$	120	48	88	$19\frac{1}{2}$
$30-60$	85	26	101	9
$60-90$	66	20	127	14
$90-120$	66	$21\frac{1}{2}$	51	16
$120-150$	67	21	78	19
$150-180$	128	43	88	$18\frac{1}{2}$
$180-210$	123	43	85	20
$210-240$	91	28	143	11
$240-270$	68	23	91	11
$270-300$	80	23	80	15
$300-330$	99	46	95	22
$330-360$	109	42	119	26

Examination of the above table shows sufficient agreement between the "median" proper motions in opposite azimuths to warrant those being taken together. When this is done the "median" proper motions are found to be as follows:—

TABLE V. Unit="001.

Azimuth.	Type II.		Type I.	
	No. of Stars.	Median Proper Motion.	No. of Stars.	Median Proper Motion.
$0^\circ-10^\circ$ and $180^\circ-190^\circ$	65	49	48	21
$10-20$ and $190-200$	88	40	61	23
$20-30$ and $200-210$	89	46	64	$16\cdot5$
$30-40$ and $210-220$	61	29	78	14
$40-50$ and $220-230$	48	32	98	10
$50-60$ and $230-240$	67	19	69	8
$60-70$ and $240-250$	42	$16\cdot5$	82	$9\cdot5$
$70-80$ and $250-260$	43	19	85	16
$80-90$ and $260-270$	49	23	51	17
$90-100$ and $270-280$	42	20	39	14
$100-110$ and $280-290$	55	23	44	$13\cdot5$
$110-120$ and $290-300$	49	23	48	15
$120-130$ and $300-310$	50	38	59	20
$130-140$ and $310-320$	55	25	60	23
$140-150$ and $320-330$	61	38	54	$21\cdot5$
$150-160$ and $330-340$	63	42	71	19
$160-170$ and $340-350$	81	36	69	20
$170-180$ and $350-360$	85	48	67	26

The above results and those of the previous table are exhibited diagrammatically. The points joined by full lines are those of the first table, and are plotted by drawing lines from O at angles 15° , 45° , etc., to the initial line, and of lengths proportional to the

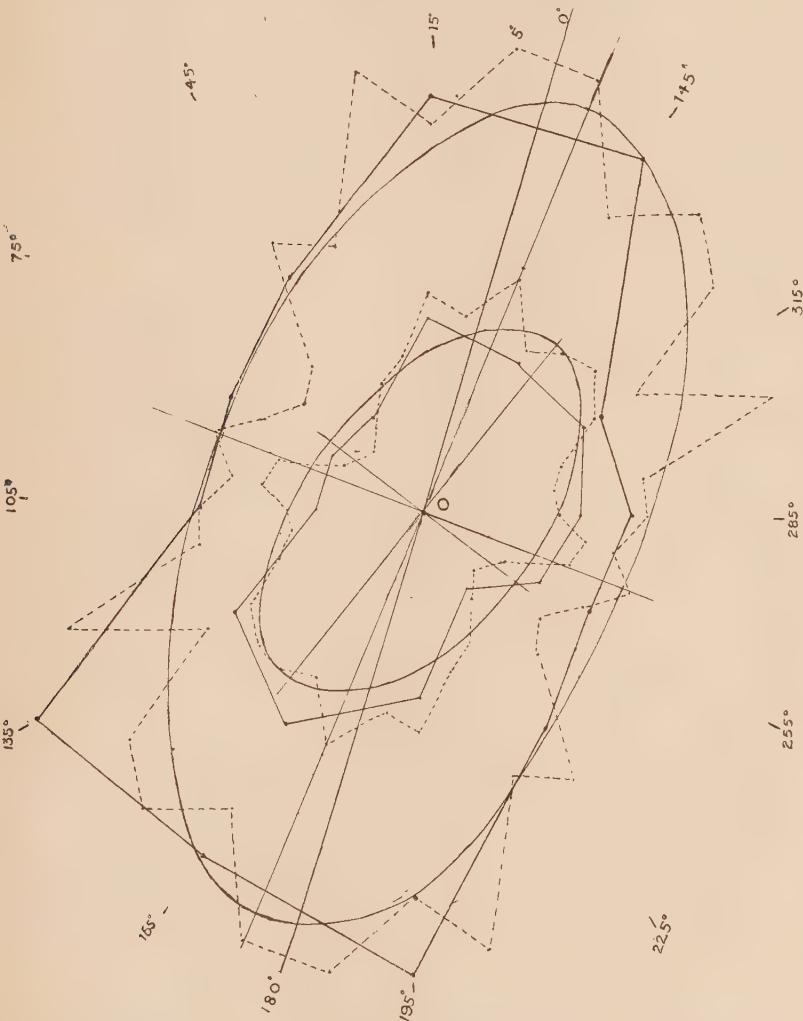


DIAGRAM 3.

median proper motion. Those points joined by dotted lines are obtained in a similar way from the second table, except that each proper motion is represented in two opposite directions.

The points which represent the median proper motions of the stars of Type II. are grouped somewhat irregularly about the larger ellipse, and those of Type I. about the smaller ellipse in the diagram.

The larger ellipse has its major axis in a direction -5° with the zero line; its major axis is 45 and minor axis 22. The smaller ellipse has its major axis in a direction -20° ; its major and minor axes are 22 and 12 respectively.

If it be assumed that the median annual proper motions of Table V. can be represented by a formula of the form

$$\alpha + \beta \cos 2(\theta + B), \text{ then we find}$$

$$\text{for Type II. } 3''.15 + 1''.31 \cos 2(\theta + 5^\circ)$$

$$\text{and for Type I. } 1''.71 + 0''.57 \cos 2(\theta + 21^\circ).$$

For stars of Type II. the maximum value is $4''.46$ in the azimuth -5° , and the minimum value $1''.84$ in the perpendicular direction. For stars of Type I. the maximum value is $2''.28$ in the azimuth -21° , and the minimum $1''.14$ in the perpendicular direction. [The minor axis of the larger ellipse in the diagram is rather too large.]

Taking the mean of the above results, the azimuths of the directions in which the cross motions are greatest are -13° and -193° . These are the points marked Q in diagram 1 and the opposite point. The R.A. of Q is $+69^\circ$ and Dec. $+58^\circ$. In accordance with Professor Schwarzschild's ellipsoidal hypothesis, the point Q should agree with the point D of diagram 2. Now D lies in the plane of OC (the apex of the solar motion) and of CA (the vertex of the stellar motions). The point Q is within 10° of this plane, whether Kapteyn's, Eddington's, Schwarzschild's, or Dyson's determinations of the positions of the apex and vertex be taken. Considering the irregularities of the proper motions, and the differences between the results found from stars of type I. and type II., this accordance is as close as can be expected. There are, however, other hypotheses which agree equally well with the facts set out in this paper. The plane of the cross motions, *i.e.* the plane of the ellipses of diagram 3, is within 20° of perpendicularity to the plane of the Milky Way. The point Q (diagram 2) which is the mean of the directions of the major axes of the two ellipses of diagram 3 is at a distance of $7\frac{1}{2}^\circ$ from the Milky Way. It follows, therefore, that the hypothesis that the proper motions of stars are greater in the direction of the Milky Way than in the perpendicular direction is sufficient to explain the results obtained here.

This examination of the resolved parts of proper motions at right angles to the direction of the solar motion is in agreement with Kapteyn's hypothesis of the existence of a vertex in the stellar motions, but does not exclude other hypotheses. Attention may be drawn to the principle underlying the method employed. Previous investigations by Kapteyn, Eddington, Schwarzschild, and Dyson have used only the *directions* of the proper motions of stars. The methods employed by them have the great merit of eliminating the *distances*. In effect, comparison is instituted

between the *linear* motions of the stars and the solar motion. In this paper the solar motion is eliminated, and resolved proper motions (angular quantities) compared with one another. The element of distance is not eliminated, and to this fact may be attributed the rather large discordances disclosed in diagram 3. But though the method here employed suffers from this disadvantage, and is also limited to a consideration of motions in one plane, it seems worth while to exhibit the results obtained by it, as such a difficult question as that of the systematic movements of the stars should be examined from as many points of view as possible.

[I have expanded the concluding paragraphs of this paper since it was presented to the Society, in accordance with a suggestion kindly made by one of the Secretaries.—F. W. D.]

Encke's Comet, 1895-1908. By O. Backlund, Associate R.A.S.

1. *The Comet's motion.*

In the *Bulletin Astronomique* I have given a brief account of my researches on the motion of Encke's comet during the period 1819-1891, and have shown that it is impossible to combine all the observations, supposing the value of the acceleration of the mean motion constant. If we represent the mean motion by the formula

$$n = n_0 + n_1 \tau ; \quad \left(\tau = \frac{t}{1200} \right)$$

the value of n_1 was very nearly constant during 1819-1858, and approximately $0''.1$. The apparitions 1861, 1865, 1868 gave a value of about $0''.08$ for n_1 , and from 1871-1895 it was only $0''.062$.* Between 1852 and 1895 the value of n' has consequently changed twice. Since 1895 there have been four apparitions, 1898, 1901, 1904, and 1908, all of which have been observed. The apparitions 1898 and 1908 belong to those in which the comet is visible only after the perihelion passage. It is well known that the comet at apparitions of this kind is very faint, and behaves, as to its brightness, quite contrary to the rule $\frac{1}{r^2 \Delta^2}$. This circumstance accounts, at least partly, for the systematic errors which characterise the post-perihelion observations. Having now finished my studies on the motion of the comet during 1895-1908, I wish to communicate some of the main results.

The first thing to be done was to form normal places from the observations. For 1895 I used the normal place derived from the observations made in January of that year (see *Vergleichung der*

* See also my memoir *Vergleichung der Theorie des Encke'schen Cometen mit den Beobachtungen, 1894-1895*.

Theorie des Encke'schen Cometen mit den Beobachtungen, 1894-1895). In 1892 Mr. Tebbutt, Windsor, N.S.W., was able to secure observations on only two nights, the comet being excessively faint. Every seventy-six years the comet and Earth resume the same configuration relative to the Sun, so that the apparition of 1898 corresponds in this respect to the apparition of 1822, when the comet was observed at Paramatta by Rümker. The two observations made at Windsor form one normal place. The apparitions of 1901 and 1904 were of the favourable types, and numerous observations were made in Europe and America; from these observations I derived seven normal places. In 1908 the apparition of the comet was of the same kind as in 1832, but probably still fainter, as it was observed at the Cape only photographically, and not visually. On five nights photographs were taken, but those of the two first nights only were well fitted for precise measurement. The two places obtained proved to be very good; one normal place was calculated by means of them. The whole number of normal places is consequently ten (in α and δ). It immediately became evident that it is impossible to satisfy all these normal places considering n_1 constant during the whole period (1895-1902), and using the same values of the planetary masses as in my preceding researches.

According to Mr. Tebbutt's remarks to his observations, the normal place of 1898 must be considered as a very weak one. I therefore first determined the system of elements which best satisfy the observations of 1895, 1901, 1904:

A.

Epoch and Osculation. 1894 Dec. 11^o, Berlin mean time.

M_0	343	21	23 ^{''} 51	} Mean Equinox 1895 ^o .
Φ_0	57	48	25 ^{''} 75	
Ω	334	44	28 ^{''} 98	
π	158	42	32 ^{''} 44	
i	12	54	17 ^{''} 18	
			4	
n_0	1074 ^{''} 09070			
n_1	+0 ^o 048600			
κ	29 ^{''} 160			

The formula for calculating M is

$$M = M_0 + n_0 t + \kappa \tau^2 + m_1 \cos 2u + m_2 \cos 4u + \dots$$

where

$$\kappa = 600 \text{ } n'$$

$$\tau = 1200 \text{ days.}$$

The periodic series converges so rapidly that it suffices to keep only the first term, the value of m_1 being

$$m_1 = -3^{''}66.$$

u is calculated from f , the true anomaly, by means of the relation

$$f = 2am \frac{2K}{\pi} u.$$

For calculating ϕ we have

$$\phi = \phi_0 - 1''.96\tau.$$

The periodic terms produced by the same force which is the cause of the secular terms in the mean motion can be omitted in π and ϕ .

The elements A represent the normal places with the following outstanding errors:—

Calcul. - Observ.		
	$\Delta\alpha \cos \delta.$	$\Delta\delta.$
1895 Jan. 12 ^o	+ 0 ^{''} 37	- 0 ^{''} 95
1898 June 13 ^o	(- 16 ^{''} 86)	(- 29 ^{''} 00)
1901 Aug. 11 ^o 5	- 4 ^{''} 14	- 2 ^{''} 21
„ 21 ^o 5	- 3 ^{''} 51	- 3 ^{''} 39
Sept. 2 ^o 0	+ 3 ^{''} 11	- 3 ^{''} 23
1904 Nov. 14 ^o 0	- 0 ^{''} 04	- 1 ^{''} 97
„ 28 ^o 0	+ 0 ^{''} 37	+ 1 ^{''} 44
Dec. 8 ^o 0	- 1 ^{''} 50	+ 2 ^{''} 68
„ 12 ^o 5	- 0 ^{''} 07	+ 1 ^{''} 69
1908 May 28 ^o 0	(- 249 ^{''} 18)	(- 123 ^{''} 20)

The residuals for 1898 and 1908 are put within brackets, because the corresponding normal places have not been used in determining the elements. Probable error of one normal place, $\pm 2''\cdot 04$.

If we now let the normal place of 1898 also contribute to the elements, it will be proper to assign only the weight $\frac{1}{4}$ in consequence of the circumstances alluded to. In this case we obtain the following elements:—

B.

M_0	343	21	23 ^{''} 31	Mean Equinox 1895 ^o 0.
ϕ	57	48	21 ^{''} 36	
δ	334	48	17 ^{''} 69	
π	158	42	24 ^{''} 55	
i	12	54	14 ^{''} 42	
n_0	1074 ^{''} 09111			
n_1	0 ^{''} 049065			

With the residuals in α and δ :

		$\Delta\alpha \cos \delta.$	C - O.	$\Delta\delta.$	
1895 Jan.	18°1	+	1''68	-	1''65
1898 June	13°0	+	1''86	-	11''04
1901 Aug.	11°5	-	4''18	-	3''42
„	21°5	-	4''82	-	4''38
Sept.	2°0	-	0''30	-	2''83
1904 Nov.	14°0	+	0''58	-	1''91
„	28°1	+	0''97	+	1''60
Dec.	8°0	-	1''39	+	2''58
„	18°5	-	1''01	+	0''97
1908 May	28°0		(-226''72)		(-122''09)

Weight $\frac{1}{4}$ *

The probable error of one normal place of the weight unity is now

$$\pm 3''\cdot 07.$$

If we compare the elements A and B, we see that the introduction of the second normal place has had no other effect than to increase the probable error. The single elements agree in both cases within the limits of their probable errors.

The amount of the probable errors in the case of elements A is satisfactory. But there is evidently a systematic difference between the outstanding errors for 1901 and 1904, the character being the same as in the corresponding preceding apparitions of the same types. These systematic differences depend mainly on the different positions of the points in the body of the comet which are observed at the several apparitions. By introducing a function $ab\psi$ with the period about twenty years in the same way as I have done for the period 1819-1858, the residuals may be considerably diminished and the number of changes of sign increased. But in consequence of the shortness of the present period a precise determination of a is impossible. I consider the elements A as the better of the two systems.

In order to compare the system A with the elements which satisfy very well the observations 1871-1895, I reproduce the latter from my memoir, *Vergleichung*, etc. The mean anomaly is corrected by 2''·02 owing to an error in the perturbations.

Elements (1871-1895).				Diff. from A.
E. and O.	1894 Dec.	11°0.		
M_0	343	21	23°09	+ 0''42
ϕ_0	57	48	14°01	+ 11''74
Ω	334	44	51°19	- 22''21
π	158	42	18°96	+ 13''48
i	12	54	24°48	- 7''30
n	1074	''09974		- 0''00904
n_1	+	0''067715		- 0''019115
κ	+	40''629		+ 11''569

* *I.e.* the real residuals are multiplied by $\frac{1}{2}$.

From this comparison it is evident that n_1 has changed considerably, about 28 per cent. The difference in n is greater than its probable error. A change in n_1 is commonly accompanied by a change in n . In the other elements the differences are not so much greater than their probable errors as to make it possible to decide whether they are real or not.

The diminution of n being established, it will be important to know when it took place. The probable error of M is about $\pm 0''.5$. Before January 18, 1895, we have employed $\kappa = 40''.629$, and after that epoch $\kappa = 29''.160$; thus we have

$$\begin{aligned} 40''.629 \tau^2 &= 0''.5 && \text{before,} \\ 29''.160 \tau^2 &= 0 && \text{after; } \end{aligned}$$

hence the diminution, if it has been sudden, can probably not have happened earlier than 133 days before or 157 days after 1895 January 18. Now the comet passed perihelion on February 5, therefore we arrive at the result that the limits are 151 before and 139 days after the perihelion, say ± 150 days ($-$ before and $+$ after perihelion).

The elements A and B do not represent the observations of 1908. If we regard the outstanding errors as coming from an error in M , we have the following relations to calculate the correction—

$$\begin{aligned} 11'.654 \Delta M + 249''.18 &= 0 \\ 5'.523 \Delta M + 123''.20 &= 0 \end{aligned}$$

which give

$$\Delta M = -21''.36.$$

If this result comes from a change in n_1 then we find

$$\begin{aligned} \Delta \kappa &= -21''.36 \\ \Delta n_1 &= -0.035600. \end{aligned}$$

This means that n_1 has again diminished nearly about the time of perihelion passage 1905 January 12. Of course this is a hypothesis, as we have not the means, as in the previous case, to verify it. We can only say that the diminution can probably not have taken place much earlier than 150 days before 1905 January 12; the limit after perihelion only the following apparitions can tell us. But in analogy with the preceding diminutions it may have happened in the neighbourhood of the perihelion.

At the time of the perihelion passage 1905 the distance between the comet and the planet Mercury reached very nearly its minimum, and the perturbations from this planet their maximum. For the last revolution (1905-1908) they go up to

$$-40'' \text{ in } M \text{ (mean anomaly), on the supposition } m_{\frac{3}{8}} = \frac{1}{9700000},$$

the value which best of all represents the observations 1819-1894. But if this value is increased by 50 per cent., i.e. if we put $m_{\frac{3}{8}}$

$$= \frac{3}{19400000}, \text{ the perturbations by Mercury (1905-1908) would be}$$

$-60''$, or a correction of $-20''$ to the calculated mean motion, which is just the quantity needed for the representation of the observation 1908. A too small value of m_8 would thus explain the difference $C - O$ 1908 as well as the hypothesis of a diminution of n_1 . Which of these two solutions of the question is the true one, only the subsequent apparitions can decide. The augmentation of m_8 is contrary to the results of the combination of the apparitions 1819-1858, which gave precisely the same value as the combination of the apparitions 1871-1891. Besides, the apparition 1894-1895 could not have been represented if we assume the greater value of m_8 without increasing n_1 . This latter supposition is very improbable. For the moment it seems, therefore, most probable that n_1 has changed, and, in analogy with the preceding changes, in the neighbourhood of perihelion. If we now ask whether the last researches have in any way advanced the solution of the question, what is the cause of the acceleration of the mean motion? the answer may be formulated as follows. The cause of the acceleration of the mean motion cannot be a resisting medium of so simple a nature as Encke supposed, for we have to consider the following three facts displayed by the latest investigations: 1st. The perturbations of higher orders than the first must, according to the formulæ of Encke, reach a certain amount, whereas the observations imply that they are necessarily very small, if not zero. 2nd. There is a periodic term of long period in M (mean anomaly), not coming from planetary perturbations, the amplitude of which for the period 1819-1859 is $4''.4$. 3rd. There have been three relatively sudden diminutions in n_1 : 1858, 1868, 1895, and very probably also in 1904.

Independently of Encke's Comet, we have evidence of the existence of matter in space (meteoric streams, etc.), which, if colliding with the comet, necessarily must modify the motion (see Seeliger, *Zusammenstösse planetarischer Massen*). But in view of the facts referred to, it seems questionable if a resisting medium, whether it be meteoric streams or regularly scattered matter, alone accounts for the irregularities of the motion of Encke's Comet. If electric forces play a part in this phenomenon, it is worth while to call attention to the coincidence of the abrupt changes in n_1 with the maximum of the activity on the surface of the Sun. Having only three or four cases to register, this of course may be a mere chance.*

We consider the case of a resisting medium. I have shown that if the resistance is proportional to $\frac{v^m}{r^n}$, where v is the velocity of the comet and r the radius vector, the observations demand

$$m + n \geq 2.$$

* In his interesting and thorough researches on the Comet 1882 II., Professor Kreutz did not detect a trace of resistance by the atmosphere of the Sun. The division of the nucleus into several parts he ascribes to electric forces developed in the comet itself at the time of the perihelion passage.

The above-mentioned fact that the perturbations, arising from the resisting medium, cannot be of higher order than the first, implies the condition

$$0 \leq m + 2n - 1 \leq 1.$$

Now the resistance cannot be independent of the velocity v . The greatest value of m if an integer is therefore 2, and this is evidently also the least, as we cannot assign a negative value to n which would be contrary to the observations. Hence $n = 0$, *i.e.* the resistance is independent of the distance from the Sun. We have consequently to do with a collision of short duration, which repeats itself at every revolution in the same distance from the Sun. By quite different considerations, however, it is proved that the origin of the acceleration is situated in the neighbourhood of perihelion. Viewed from this standpoint, the comet crosses a meteoric swarm in the neighbourhood of perihelion. The observed diminutions of the acceleration of the mean motion might then be a consequence of a diminution of the density of the meteoric ring.

If we denote the resisting force by $K \frac{v^m}{r^n}$, where v and r signify respectively the velocity and radius vector and K a function not only of the elements but also of the nature of the comet itself, then a change in this latter may also call forth a change in n_1 . But the observations do not indicate any considerable change in the appearance of the comet after the diminution of n_1 . If the supposed diminution in 1904 is real, then n_1 is at present very small, only the ninth part of what it was before 1858; a corresponding change in the comet itself is not likely to have been unnoticeable. We must therefore consider the diminution of the density of the resisting medium as more probable.

Nowadays, nobody will seriously deny that there are electric or electro-magnetic forces in play on the comets. The ejections and the different types of tails cannot, as Bredichin showed, be due alone to the pressure of light. But the present state of our knowledge as to the electric forces tangential to the orbital motion is so limited that every attempt to explain the irregularities of the motion of Encke's Comet exclusively from this point of view will prove futile. The only indication in this direction is the coincidence of the diminutions of n_1 with maxima of activity on the surface of the Sun, and this might be purely accidental.

The conclusion is: if we exclude every possibility of electric tangential forces, and consequently also regard the coincidence mentioned as a purely accidental one, then we must adopt the resisting medium in the form of a meteoric ring as the cause of the acceleration of the mean motion, the diminution of n_1 being a measure of the decrease of the density of the medium. The comet crosses the ring in the neighbourhood of perihelion. If, on the contrary, the coincidence of the variation of n_1 with maxima of solar activity is supposed not to be accidental, and tangential electric forces are admitted, then our problem is actually an indeterminate one.

2. *On the variation of the brightness of the Comet.*

Encke's Comet exhibits other peculiarities in addition to those connected with its motion: I mean the fluctuations of its brightness. In consequence of its small perihelion distance and the short time of revolution, the cometary mass is very much exposed to dissolution by the Sun at intervals of $3\frac{1}{2}$ years. It seems, then, natural that loss of mass should be connected with loss of brightness. Only a relatively small diminution of light is indicated during the period 1819-1904. At the brightest apparitions, 1828 and 1838, when it was visible to the naked eye, the brightness of the comet was estimated as equal to that of a star of the 5th magnitude. In 1881 I could see the comet with the naked eye. At the apparitions of 1901 and 1904 it was estimated of the 6th magnitude. The general loss of brightness during the period 1819-1904 should thus have been about one magnitude, from the 5th to the 6th. Leaving the different aspects of the comet at different epochs, I wish to call the attention to the singular fact that after perihelion it is considerably fainter than before perihelion.

Among the attempts to explain this phenomenon, the weakest seems to me to be that which is based on the sun-spots. According to this explanation, the maximum of brightness coincides with the maximum of sun-spots, minimum of brightness with minimum of sun-spots. That this does not hold good, we can at once see by comparing observations before and after perihelion at the same apparition. We take, for instance, the apparition in 1875, March 17, before perihelion, and May 7, after perihelion; we have respectively:

	March 17.	May 7.
Log r	9.8672	9.8308
„ Δ	0.1749	9.7413
„ $\frac{1}{r^2 \Delta^2}$	9.9158	0.8558
„ $\frac{1}{r^4 \Delta^2}$	0.1814	1.1942

From these numbers it follows immediately that on May 7 the conditions for visibility are much better than on March 17. But on March 17 the comet was an easily observed object, while on May 7 it is recorded as excessively faint by Mr. Tebbutt at Windsor. In 1822 and 1832, after perihelion, the comet was difficult to observe; these apparitions correspond very nearly to the apparitions of 1898 and 1908 respectively, when it evidently was still fainter. In 1898 Mr. Tebbutt of Windsor observed the comet on June 11 and 15, and remarks that it was utterly faint. On June 25, when the comet had approached nearer to the Earth ($\Delta = 0.3$), it was invisible, the night being clear and moonless. On the following nights the comet became again visible, but too faint to be observed with the micrometer.

The hypothesis that the particles of the comet are orientated, and respectively move in nearly one plane, accounts on an average for the variation of brightness after perihelion. Of course there are details which require supplementary hypotheses. It would exceed the limits of this paper to reproduce the details of my investigation on this subject; I therefore confine myself to stating the position of the comet's plane on June 23, 1898. Inclination to the ecliptic, 39° ; the angle between nodal line on the ecliptic and the major axis of the comet's orbit 113° , the axis supposed to be in the plane of the ecliptic.

The position of the plane, supposed to move parallel to itself, can be determined for any time relatively to the initial plane by means of the formula

$$M = (z - z_0) \frac{\cot i}{\sin f_0} + r \cos f - r_0 \cos f_0,$$

where M is the length intercepted on the major axis of the orbit by the two planes, r the radius vector, f the true anomaly of the comet, i the inclination of the cometary plane to the ecliptic, and f_0 the angle between the line of nodes and the major axis. The initial plane is determined by the condition that it passed through Sun-Comet-Earth on 1898 June 23. Then the illumination by the Sun is the least possible, and the comet being nearly a plane containing the line of sight, must be nearly invisible from the Earth. At intervals of seventy-six years the same configuration repeats itself. At intervals of ten years, not far from the epoch, the above condition is also approximately fulfilled. The apparitions at which the comet passes through perihelion at the end of April and in May are remarkable for the minimum of brightness of the comet. Before perihelion the Sun and Earth can never be simultaneously in plane of the comet. So far, and in accordance with the initial conditions, the observed variations are tolerably represented; it is easy, by help of the elements of the plane, to calculate the angles subtended at the Sun and Earth in the various positions of the plane. To this manner of regarding the question there are important objections. The comet appears in the cases mentioned as a circular patch. If the comet is really flat and its plane moves parallel to itself, new difficulties arise concerning the resistance, because it is not easy to understand how this plane, by crossing a meteoric swarm, would produce the effect displayed by the observations. Then the stability of the arrangement of motion in one plane must be proved. Schiaparelli, Charlier, Picart, and Tisserand have studied the motion in a cometary system. Charlier and Picart arrived at interesting results, but they are of very little use here, because they suppose the cometary cluster spherical and the motion of the comet circular. The only use we can make is that, for example, Schiaparelli's formulæ (Tisserand, *Mécanique céleste*, t. iv. p. 259) show that the action of the Sun produces tides, in consequence of which the comet is

deformed, as it probably yields very easily to the perturbative force. The combined effect of Sun and Earth may then really be such as to flatten the comet, at least for some time. Then, however, it is difficult to see why the comet's plane continues to move parallel to itself; on the contrary, it seems more probable that it continually passes through the Sun, which again demands that similar variations of brightness take place before and after perihelion, contrary to the observations.

It is not impossible that here also there are electric forces in play, but we are completely ignorant how such forces act so as to make the comet almost invisible just at the minimum distance from Earth (0.3) and simultaneously at a distance from the Sun < 1 . Thus the interesting question of the extraordinary behaviour of the variation of the comet's brightness must as yet be left open.

On a possible connection between Encke's Comet and the Comet discovered by Professor Wolf in Heidelberg on Dec. 7, 1907.

In searching for Encke's Comet, Professor Wolf found on 1907 Dec. 7 an object very near the place given by the ephemerides which he supposed to be Encke's Comet. In January 1909 he succeeded in securing 7 observations, showing by the character of the deviations from the ephemeris that it could not be Encke's Comet, which was ultimately proved by Mr. Ebell.

From the seven observations in January 1908 Mr. Ebell calculated a parabolic orbit; later on Mr. Matkewitch, by means of the method of least squares, deduced the most probable elements. The deviation from the parabola in the sense of an ellipse is very small, the difference of the excentricity from unity being less than its probable error. I reproduce here the elements only to tenths of a minute. For the sake of comparison I give also the elements of Encke's Comet.

Comet Wolf, 1907.			Encke's Comet.		
T	1907 Dec. 6 ^h 03 ^m 81 ^s 09.		M.T. Berlin.		
<i>e</i>	0.9999388			0.83641	
π	356°	32' 8"	} Mean Equinox 1908.0	159°	5' 6"
Ω	317	6.1		334	30.3
<i>i</i>	10	26.7		12	36.7
$\log q$	0.58445		$\log a(1+e)$	0.61314	

The elements Ω and *i* resemble each other; the longitude of perihelion of Comet Wolf differs from the longitude of aphelion of Encke's Comet by 17°. The least distance between the two orbits is 0.16. It is difficult to resist the temptation to suppose that Encke's Comet and Comet Wolf formerly have belonged to one and the same family of comets. To prove this rigorously is of course not possible, but I may be allowed to give my investigations on this point.

All the known periodical comets of short period, Encke's Comet excepted, are considered as captured by Jupiter (see Callandreau, "Sur la théorie des Comètes," *Annales de l'Observatoire de Paris*, 1892). A necessary condition is: $3.14 > a > 2.60$, where $2a$ is the major axis in the orbit as transformed by Jupiter. The lower limit is $\frac{a'}{2} = 2.60$, where $2a' = \text{Jupiter's mean distance}$.

The value $a = 2.2$ for Encke's Comet is consequently outside the condition 2.6 for capture. The lower limit can be depressed by about 0.13 if the comet enters the sphere of activity when Jupiter is in perihelion, for we have in this case $\frac{r'}{2} = 2.47$ as the lower limit.

Suppose now that a comet moves in an orbit with the same π , Ω , i which belong to the orbit of Comet Wolf, but with a slightly different perihelion distance; further, we imagine a comet having the same elements as Comet Encke except the major axis, for which we assume $a = 2.4774$. What value must be assigned to q_0 (perihelion distance of the first comet) in order to satisfy the invariant

$$\frac{1}{a} + \frac{2}{r'} \sqrt{\frac{p}{r'}} \cos i = 0.55113 = \frac{1}{a_0} + \frac{2}{r'} \sqrt{\frac{p_0}{r'}} \cos i_0?$$

The suffix $_0$ refers to the first comet. We put of course $a_0 = \infty$. By means of the value i_0 , from Comet Wolf, the second equation furnishes the value

$$\log \frac{p_0}{2} = \log q_0 = 0.67700.$$

If we suppose that the perihelion of the first comet coincides with the perihelion of Jupiter,* it is easily shown that it can enter the sphere of activity of Jupiter. For smaller values of q_0 , e.g. for $q_0 = 0.62000$, the comet can evidently also be captured; corresponding to this value, the invariant is given by $\log (\text{inv.}) = 9.72277$, whence $\log a = 0.42590$. For all intermediate values the condition of capture is fulfilled. In the following I shall treat only the first case.

We assume, then, that the capture took place for $\log q_0 = 0.67700$, so that the value of a in the new elliptic orbit is $\log a = 0.39940$, and the corresponding value of $n = 9.10''$. For the present value of n of Encke's Comet $5n - 18n'$ is a small quantity, from which we conclude the configuration Jupiter-Comet-Sun is repeated very nearly at intervals of 59 years.

The minimum distance between Jupiter and Encke's Comet is 0.9. In 1820 the comet passed Jupiter nearly at this distance; during the revolution 1819-1822 the perturbation by this planet amounted to $-7''$ in n . The next approaches followed in 1843 and 1866, the perturbations during the corresponding revolutions being

* This supposition will be justified below.

positive and the sum about $+7''$. In 1879 the configuration was approximately the same as in 1820 and the perturbations again about $-7''$, and so on. For the mean motion $910''$ the circumstances are analogous. $6n - 18n'$ is now a small quantity (though greater than $5n - 18n'$ in the former case), and the perturbations may possibly have followed in the same order, though the intervals are modified. For the sake of simplicity we will admit that the capture took place, so that in the two following approaches the perturbations in n by Jupiter were positive; as the corresponding distance from Jupiter at the approach is essentially less than in the present case of Encke's Comet, the perturbations have been about ten times greater. In this way the comet cannot have been exposed to recapture at its returning to the critical position, the minimum distance being greater than immediately after the capture. In some 120 years it should for the second time be in the critical position again, but owing to the effect of the "resisting medium" it is definitely saved. In fact the acceleration due to a tangential force opposite to the direction of the motion has then in the course of time increased n and diminished a to their present values.

Having thus traced the way in which Encke's Comet can possibly have been captured and brought to move in its present orbit, it may be of some interest to have a general idea of when the capture happened.

The motion of the comet's perihelion in consequence of the action of Jupiter is expressed by the following well-known formula :

$$\frac{d\pi}{dt} = -a \cos^2 \phi R + (a \cos^2 \phi + r) \sin f \cdot S + 2 \sin^2 \frac{i}{2} \frac{d\Omega}{dt}.$$

The last term, as being small in comparison with the other, may be omitted. R and S are expressed as follows :

$$R = N \left(\frac{\xi - r}{\Delta^3} - \frac{1}{r'^3} \right)$$

$$S = -N \eta \left(\frac{1}{\Delta^3} - \frac{1}{r'^3} \right).$$

N is a positive constant proportional to the mass of Jupiter. When the comet is in the neighbourhood of aphelion we have $\xi > r$, $\xi - r$ positive, and $\cos f$ is negative; therefore R is always positive in the neighbourhood of aphelion (before or after). η goes through zero not far from aphelion, $\sin f$ also goes through zero in aphelion. The second term is therefore less than the first. It is then mainly the first term at the three approaches during the period of 59 years which determine the motion $\frac{d\pi}{dt}$, which is always direct. The calculation of the perturbations 1819-1908 gives $+0.65$ as the present yearly motion of perihelion of Encke's Comet. Immediately after the capture it can have been, as a

direct calculation has shown, about 10 times greater. From considerations which it is not necessary to report here, I have admitted a rough approximation $\frac{6.5 + 0.65}{2} = 3.57$ for the mean yearly motion

of the perihelion from the capture up to present time.

At the present time the aphelion of Encke's Comet is 32° behind that of Jupiter. The mean motion of Jupiter's perihelion is 0.1 ; then we have for determining of T the time elapsed since the capture:

$$3.57T = 2n\pi + 328^\circ + 0.1T.$$

If we put $n = 0$ we obtain

$$T = 5672 \text{ years.}$$

On the other hand we may determine how long time it takes to bring the mean motion from $910''$ up to $1075''$, supposing the acceleration of the mean motion to have had the mean value of $0''.1$ in 1200 days.

We find

$$0''.17 = 165$$

$$\tau = 1650 \text{ periods of 1200 days (or 3.3 years approx.)}$$

$$T = 5423 \text{ years,}$$

which is accidentally in very good agreement with the former value, and shows that on an average the tangential force has been of the same magnitude as during 1819-1858.

The epoch when the comet was captured having been found, we are in a position to calculate the longitude of Jupiter's perihelion at that epoch. This we obtain from $5662 \times 0''.1$, or about 9° . The present longitude is 11° , at the epoch in question it was consequently 2° . The longitude of the perihelion of Comet Wolf is 356° . This is the reason for assuming the coincidence of the perihelion of Comet Encke with the perihelion of Jupiter before the capture. If we put $n = 1$ in the general formula we find

$$T = 11292 \text{ years,}$$

and for the acceleration

$$n_1 = 0''.05,$$

which seems less probable.

It seems, then, as if 5700 years were an upper limit for the time of capture.

This result must, however, be accepted with great reserve. (1) It is not proved that the comet has really been captured by Jupiter, only that the capture may have been possible. (2) The motion of perihelion is only a rough approximation. (3) Though the acceleration during 1819-1908 has changed only by diminishing, it is not impossible that it has been less than $0''.1$ in former times.

More trustworthy results can perhaps be obtained by calculating the path of the comet through Jupiter's sphere of activity, but without hypothesis this method will not accomplish our object; still such a work may possess some interest.

Pulkovo :
1910 February.

The Envelopes of Comet Morehouse (1908 c).
By A. S. Eddington, M.A., M.Sc.

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§ 1. One of the most striking features of the photographs of Comet Morehouse, obtained with the 30-inch reflector at the Royal Observatory, Greenwich, is the clear definition of the parabolic envelopes, which often appear in and near the head of the comet. The very rapid changes of these envelopes form an important subject for study, and one that may be expected to throw light on the phenomena involved in the formation of the tail. When closely examined, the constitution of these envelopes is a much more perplexing problem than would at first sight appear; and the present paper does not claim to establish any definite conclusion. The great difficulty is their almost instantaneous formation, which seems to require the existence of a force of solar repulsion enormously greater than that generally accepted. I raised this question briefly in *Proc. Roy. Inst.*, 1909 March 26, and now give a fuller discussion.

The present investigation forms part of a general discussion of the Greenwich photographs of the comet, which I have undertaken in conjunction with Mr. Davidson, and in this paper I am indebted to him for assistance in many ways. In view of the return of Halley's Comet, which may present opportunities of acquiring evidence on the doubtful points, it has seemed best not

to delay the publication of this until the whole work was completed.

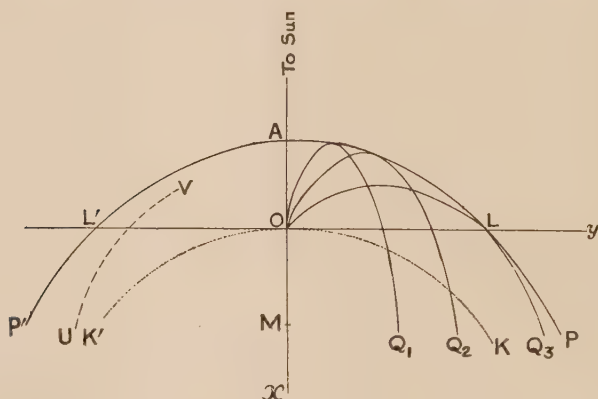
§ 2. *Measurement of the Photographs.*—The photographs were measured in a position-micrometer with a low-power eyepiece. The position of the radius vector from the Sun was computed, and the plate oriented so that it coincided with the direction of motion of the microscope by the micrometer-screw. The diaphragm in the microscope was ruled with two fine lines intersecting at right angles, each inclined at 45° to the direction of motion. This arrangement was originally adopted in order to take advantage of the property of the parabola,—“Any pair of tangents at right angles to one another meet on the directrix.” The point of intersection of the lines could thus be placed successively on the focus (nucleus), vertex and directrix, giving two independent determinations of the focal length of the envelope. But in addition, this diaphragm proved to be a very convenient device for measuring the two rectangular coordinates with one micrometer-screw; if a point is brought successively under the two lines, half the sum of the readings gives its x coordinate, and half the difference gives its y coordinate.

In almost every case the nucleus showed as a well-defined stellar point, and the error of setting on the nucleus was practically nil.

§ 3. *Fountain-Theory of Envelopes.*—It is well known that when a great number of particles are projected from a point with equal velocities in all directions under gravity, the envelope of their paths is a paraboloid having the point of projection as focus. There seems to be little doubt that cometary envelopes are formed in an analogous manner. In this case the repulsion from the Sun takes the place of gravity, the particles are supposed to be projected from the nucleus with equal velocities in all directions in the hemisphere towards the Sun (those in the hemisphere away from the Sun have no part in forming the envelope), and the envelope is a paraboloid having the nucleus as focus and forming a kind of dome over it. In the diagram, OQ_1 , OQ_2 and OQ_3 are the paths of individual particles, and all touch internally the parabola PAP' , which is the envelope. This theory, which we shall call the *fountain-theory*, was adopted by Bessel, Bredichin, Bond, and others. It will be noticed that according to this theory at successive instants different particles form the envelope; the change of position of an envelope is not equivalent to the corresponding material displacement. In a later section I shall consider the evidence for this theory of the envelopes; but meanwhile it may be stated that the difficulties in the way of any other explanation appear to be overwhelming.

The envelopes, which appear in the photographs, are for the most part sharply-defined fine curves, and differ rather markedly from the appearances to which the term is usually applied in other comets, although they are probably analogous. It is possible to use the word “envelope” in two rather different senses, either as

a boundary beyond which little or no material is projected, or as a surface of exceptional density of the material. It is the latter feature that particularly characterises the envelopes of Comet Morehouse, whereas in the case of other comets, such as Donati, Coggia, 1910 *a*, in which the envelopes have been particularly noticed, the term has been used in the former sense to denote a boundary with little or no increase in density of the material. Both kinds of envelope are accounted for by the fountain-theory; but it would seem that in Comet Morehouse each fine envelope corresponds to a distinct and separate explosion, whilst in the



other comets the explosions were so numerous that only a general average effect was observed.

§ 4. *Transitory Character of the Envelopes.*—It is for this reason that we were able to observe in Comet Morehouse, what I believe had not been noticed in any previous case, the extremely transitory character of the individual envelopes. In Table I. are given measures of envelopes on October 27 and 30, these being dates when the most abundant data are available, but the same thing can be seen on any night when envelopes were shown. Where possible, two independent measures made respectively at the apex and latus rectum of the envelope are given. It will be seen that the envelope always begins to collapse immediately after its first formation, and shrinks continually. *I know of no case where an envelope expands.* After three or four hours it has completely degenerated, the part between the nucleus and the Sun being lost in the uprushing material for new envelopes, whilst that behind the nucleus mingles with the tail. It is also remarkable that the envelope becomes more strongly defined as it contracts.

TABLE I.

Focal Lengths of Envelopes, showing the rapidity with which the force of the explosion declines.

No. of Envelope.	Measures at	Time of Photograph, 1908 October 27.									
		6 ^h 41 ^m .	7 ^h 5 ^m .	7 ^h 23 ^m .	7 ^h 44 ^m .	8 ^h 4 ^m .	8 ^h 23 ^m .	9 ^h 3 ^m .	9 ^h 32 ^m .	10 ^h 2 ^m .	10 ^h 28 ^m .
1	A	'80	'74	'66	'47	'44	'39	'39	'33	'30	'...
	L	'76	'62	'57	'49	'42	'36	'33	'30	'25	...
2	A	1'46	1'30	'97	'86	'60	...	...	...	...	...
	L	1'34	1'09	1'02	'86	'61	'52	'46	...	...	...
4	A	...	...	...	...	'97	'74	'67	'64	...	...
	L	...	...	...	...	...	...	...	'68	'64	'54
5	A	2'3	2'2	1'56	...	1'28	1'06	'99	...	...	...
	L	...	...	...	...	...	1'04	'94	...	...	...
6	A	...	...	...	...	...	1'90	1'62	1'38	1'22	1'15
	L	...	...	...	...	...	...	1'48	1'38	1'12	...
1908 October 30.											
		6 ^h 21 ^m .	6 ^h 37 ^m .	7 ^h 2 ^m .	7 ^h 21 ^m .	7 ^h 47 ^m .	8 ^h 4 ^m .	8 ^h 19 ^m .	8 ^h 52 ^m .	9 ^h 19 ^m .	
1	A	1'26	1'16	'92	'63	'...	'49	'...	'...	'...	
	L	1'16	1'07	'85	'74	'63	'57	'39	...	...	
2	A	'60	'57	'44	'38	...	...	...	...	...	
	L	'62	'55	'45	'40	'34	'26	'22	..	...	
3	A	1'66	1'52	1'34	1'22	'99	'88	'81	'66	'50	
	L	...	1'48	1'34	1'27	1'04	'89	'76	'65	'46	
4	A	...	...	...	...	1'55	1'50	1'41	'90	...	
	L	...	...	...	...	1'69	1'64	1'29	'90	'65	

It seems clear that each envelope corresponds to a single outburst, strongest at first, *i.e.* projecting matter with greatest velocity, and gradually declining in force, though not necessarily in the volume of matter emitted, the whole outburst lasting three or four hours. Several outbursts are generally taking place simultaneously, presumably at different points of the nucleus. The question may be raised whether all the matter experiences the same repulsive force from the Sun. Where two or more envelopes exist simultaneously, it might be thought that they correspond to matter undergoing different amounts of repulsion, as in Bredichin's theory of the different tails. But I think that the size of the envelope depends chiefly on its age; on any day the envelopes form successively at more or less the same height, and as each contracts, another is formed outside; thus, when several envelopes exist simultaneously, they may be regarded as showing successive stages of similar outbursts. Nevertheless, the envelopes on different

nights often vary very greatly in size and general character, and it is probable that there are large variations in the repulsive force.

§ 5. *Time of formation of an Envelope.*—When an outburst takes place, some time must elapse before the ejected material reaches the required position and forms the envelope. Further, the envelope cannot all be formed at the same instant; the part at the apex forms first, and the envelope gradually spreads, and extends further and further away from the Sun.

Let V be the velocity of ejection, g the accelerating force of repulsion from the Sun.

Take the origin at the nucleus, and the axis of x along the radius vector away from the Sun (see diagram, p. 444).

After a time t , the position of a particle ejected at an angle $(180^\circ - \alpha)$ to Ox , is

$$\begin{aligned}x &= -Vt \cos \alpha + \frac{1}{2}gt^2 \\y &= Vt \sin \alpha,\end{aligned}$$

whence, eliminating α ,

$$\frac{1}{4}g^2t^4 - (V^2 + gx)t^2 + x^2 + y^2 = 0 \quad . \quad . \quad (1)$$

and the condition for real roots is

$$y^2 < \frac{2V^2}{g} \left(x + \frac{V^2}{2g} \right).$$

Thus the envelope is the parabola

$$y^2 = 4a(x + a), \quad \text{where} \quad a = \frac{V^2}{2g}.$$

(In the figure this parabola is $PLAL'P'$ and $a = OA$.)

The time taken to reach any point on the envelope is derived from equation (1). Remembering that for points on the envelope the equation has equal roots, we obtain at once

$$\frac{1}{2}gt^2 = \sqrt{(x^2 + y^2)} = x + 2a \quad . \quad . \quad (2)$$

and
$$t = \sqrt{\frac{2}{g}} \cdot \sqrt[4]{(x^2 + y^2)} = \sqrt{\frac{2}{g}} \sqrt{x + 2a} \quad . \quad . \quad (3)$$

The time is thus equal to that taken by a particle to fall from rest on the directrix to the point considered, under the repulsive force.

For example, putting $x = -a$ for the apex A , and $x = 0$ for the latus rectum L . The time from the nucleus to the apex

$$= \sqrt{\frac{2a}{g}}; \text{ and the time to the latus rectum is } \sqrt{\frac{2a}{g}} \times \sqrt{2}.$$

We cannot observe either of these directly; but we can, by noticing the growth of the envelope on successive photographs, estimate their difference, *i.e.* how long after the first appearance of the envelope at A it has spread as far as L and L' . This gives

$$\sqrt{\frac{2a}{g}} (\sqrt{2} - 1); \text{ and, as we can measure } a, g \text{ can be found.}$$

In general, the paraboloidal envelope has its axis inclined to the plane of the photograph, so that what we see on the photograph is not a central section of the paraboloid. It can be shown that the apparent outline will always be a parabola, having the nucleus as focus. The focal length a , however, is not foreshortened, but is greater in the parabola than in the paraboloid in the ratio $\sec I$, where I is the inclination of the axis of the paraboloid to the plane of the photograph. For most purposes it is sufficient to ignore motions in the line of sight, considering only the particles projected in the plane of the photograph, and the component of the Sun's repulsion resolved in that plane. This leads to the same result. But it is useful to remember that the apex of the parabola does not correspond to the apex of the paraboloid; for instance, when I is 45° , the apex of the parabola is a point on the focal plane of the paraboloid. In the case of Comet Morehouse, during the period under consideration, I was never less than 43° , and sometimes as much as 60° . We naturally think of the apex of an envelope shown in the photograph as lying between the nucleus and the Sun, but, owing to the inclination, this was not usually the case. We see practically nothing in these photographs of the paraboloid between the nucleus and the Sun, that part having nothing to do with the parabolic outline which is the two-dimensional appearance of the envelope.

§ 6. It is found that the photographs of Comet Morehouse indicate generally a remarkably rapid formation of the envelope. Very soon after the first appearance of envelope formation at the vertex, the whole envelope, extending some distance beyond the latus rectum, appears to be complete. It is not very easy to put in a convincing form the evidence on which this assertion rests. The extremity of an envelope is seldom abruptly defined, and its apparent extent depends on the excellence of the photograph; further, the envelopes tend to become denser and more clearly defined as they contract, and from this cause may be traced farther. I give, however, a few instances in which the evidence can be definitely stated, and appears to me unmistakable; these are supported by a great deal of less precise confirmatory evidence.

1. Oct. 18. —An envelope on the fore side of the comet (*i.e.* in the direction in which it is moving). On plate 3173 ($7^h 58^m$) there is no sign of this envelope; a little hazy material is seen where the vertex would be, but there is no ridge or distinct boundary. On plate 3174, 57 minutes later, the envelope has formed; it is very distinct, fine and regular, one of the most perfect examples of an envelope that I have come across. It extends clearly beyond the nucleus to $x=1'1$, and is traceable faintly to about $x=2'$. On plate 3175, 55 minutes later, the envelope is less sharply defined, but extends clearly to $x=1'9$, and can be traced farther. The values of a (focal length) are, on plate 3174, $0'56$; on plate 3175, $0'25$.

This evidence of extremely rapid formation is the more

convincing as, of the three photographs (each having 10 minutes' exposure), 3173 is much the densest and shows the most detail generally, and 3174 is considerably better than 3175. Thus the absence of the envelope on 3173 cannot possibly be attributed to inferiority of the photograph.

2. Oct. 15.—Envelope on rear side.

Plate 3167. $7^h 3^m$, invisible.

3168. $7^h 24^m$, faint, but formed as far as $x = 1'.2$.

3170. $8^h 20^m$, well defined, extends at least to $x = 5'.6$.

Values of a are $0'.72$ and $0'.48$ on 3168 and 3170 respectively. The photographs are of practically equal density and definition.

3. Oct. 15.—Another envelope on rear side.

Plate 3170. $8^h 20^m$, envelope just formed at vertex, $a = 1'.18$.

3171. $9^h 24^m$, envelope extends at least $2'.5$ beyond its latus rectum, $a = 0'.95$.

4. Oct. 27.—Envelope chiefly on fore side, but also extending some distance on rear side.

Plate 3203. $8^h 4^m$, haze near where apex of envelope would be, but no indication of a definite boundary.

3204. $8^h 23^m$, boundary distinct; the envelope seems to be just forming at the apex $a = 1'.90$.

3206. $9^h 3^m$, envelope clearly seen as a ridge, and extends nearly to the latus rectum, $a = 1'.55$.

3207. $9^h 32^m$, envelope fully developed and extends just beyond the latus rectum, $a = 1'.38$.

5. Oct. 30.—Envelope chiefly on rear side; rather indistinct.

3220. $7^h 21^m$, invisible.

3221. $7^h 47^m$, envelope formed for a short arc at vertex, $a = 1'.5$.

3222. $8^h 4^m$, envelope nearly reaches latus rectum, $a = 1'.45$.

3223. $8^h 19^m$, envelope reaches beyond latus rectum, $a = 1'.42$.

6. Oct. 1.—Envelope on rear side.

3127. $7^h 53^m$, quite invisible.

3128. $8^h 35^m$, extends $2'$ beyond the nucleus; for this envelope, a is very small, about $0'.1$.

The exposures were 30^m and 15^m respectively, and it is just possible that the long exposure of 3127 may render the envelope too indistinct on that plate to be visible, but I think that is unlikely.

§ 7. *Calculation of the Repulsion.*—The first example given above is the most decisive. Let us examine to what lower limit of the value of g it leads. a is variable, but we may take $a = 0.7$ as being probably about the mean value of a between plates 3173 and 3174. If we assume that the envelope began to form immediately after 3173 was exposed, and that on 3174 it extended to $x = 1.1$ (ignoring the fainter continuation), the resulting value of g will certainly be a lower limit. If t be the time from the nucleus to the apex in hours, equation (2) gives

$$\begin{aligned}\frac{1}{2}gt^2 &= 0.7 \\ \frac{1}{2}g\left(t + \frac{19}{20}\right)^2 &= 1.1 + 2 \times 0.7 = 2.5,\end{aligned}$$

whence eliminating t , $g = 1.23$ per hour per hour; converting into kilometres and allowing for the inclination of the radius vector to the plane of the plate, the full force of solar repulsion is not less than 81,500 kilometres per hour per hour. But the solar gravitation is only 34.6 km./hr.² on this date. Thus the repulsion appears to be more than 2300 times the ordinary gravitational attraction. The other examples lead to the following lower limits for g , and μ (the ratio of the solar repulsion to the gravitational attraction).

Ex. 2.	(Oct. 15)	$g > 2.0$	per hour per hour;	$\mu > 4,000$
3.	(Oct. 15)	$g > 1.8$	„ „	$\mu > 3,500$
4.	(Oct. 27)	$g > 0.11$	„ „	$\mu > 180$
5.	(Oct. 30)	$g > 4.1$	„ „	$\mu > 6,500$
6.	(Oct. 1)	$g > 6.1$	„ „	$\mu > 19,000$

These values are of course much greater than the values ordinarily found. In Bredichin's researches μ was not greater than 36; subsequent discussions of observations by Jaegermann and others have led to values of μ up to about 80. Direct measurement of the motion of the tail particles of Comet Morehouse leads to values of μ ranging under normal conditions up to rather more than 100; and on October 1 and 2, when the motion was quite exceptionally rapid, μ may have been as great as 800. But the above results seem to show that the repulsive forces acting on the envelope-material are generally much greater still, and it will be necessary to carefully consider whether there is any possible escape from this conclusion.

§ 8. *Second Method.*—There is another method of testing the rapid formation of the envelopes, which leads to more precise numerical results, provided the simple theoretical conditions we have stated are rigorously fulfilled. If the eruption of material is not in a steady state, the form of the envelope will no longer be a paraboloid but will be modified. It has been shown that as we pass along the envelope from the apex, the matter forming the envelope at the instant has taken successively longer intervals to travel from the nucleus to the present position, and therefore was emitted at correspondingly earlier stages of the explosion. But as

the force of the explosion was greatest at first and gradually declined, the earlier the time of emission the larger will be the paraboloid to which this portion of the envelope belongs. If we still write the equation of the envelope

$$y^2 = 4a(x + a)$$

$a = \frac{v^2}{2g}$ is variable, that value of v being taken which corresponds to the instant at which the matter now at (x, y) left the nucleus. The successive elements of the curve may thus be considered to form parts of parabolas, of which the focal length gradually increases with x . The shape will in fact be hyperbolical.

This is actually found to be the case on measuring the photographs. I have measured a great many envelopes to determine their precise shape, and find that along any envelope, except occasionally between the apex and latus rectum, a steadily increases with x . I select one or two examples to make this clear.

TABLE II.

Hyperbolic shape of envelopes, shown by change of the value of a at different parts of the curve.

(Unit = 2'.5.)

Plate 3139. Oct. 2.		Plate 3164. Oct. 14.		Plate 3201. Oct. 27.		Plate 3229. Nov. 3.	
x	a	x	a	x	a	x	a
- '12	'326	+ '28	'083	+ '08	'402	+ '45	'204
+ '41	'432	1'17	'132	'86	'409	'86	'218
+ 1'34	'597	1'81	'164	2'26	'441	1'52	'223
		4'05	'252				

Between the vertex and latus rectum the value of a sometimes decreases slightly, as Table I. shows, but this is not necessarily a real effect. The envelopes at the vertex are always close together, very dense and often confused. In these circumstances, seeing that we are measuring an outer limit, there may be a tendency to exaggerate the size near the vertex. The measures near and beyond the latus rectum seem to be much superior in accuracy, as the envelope is there clearly separated from those immediately inside or outside it. It has already been pointed out that the part of the *parabola* between the vertex and latus rectum does not correspond to the part of the *paraboloid* between the nucleus and the Sun, so that it is unlikely that the shape of this part of the envelope follows a different law from the rest.

Now if, taking the same envelope on two different photographs, we select two points, one on each of the photographs, for which a is the same, presumably the matter at these two points left the nucleus with the same velocity V , and therefore at the same instant. This is sufficient to enable us to determine g . Thus, on September 17, there is a very sharply defined envelope on the

fore side of the comet, for which I obtained the following measures of arbitrary points on the curve.

1908 Sept. 17 (Unit=2'.5).				
Plate and Time.	x .	y .	$\sqrt[4]{x^2+y^2}$.	a .
3094	- '043	'192	'44	'120
8 ^h 12 ^m	+ '290	'543	'78	'163
	1'107	1'038	1'23	'206
3095	+ '017	'159	'40	'072
9 ^h 6 ^m	'450	'528	'83	'122
	1'021	'831	1'15	'148
	2'593	1'585	1'74	'223
	3'616	1'965	2'03	'250
3096	+ '179	'243	'55	'061
10 ^h 4 ^m	'868	'636	1'04	'104
	1'802	1'019	1'44	'134
	4'067	1'974	2'13	'227

Now equation (3) shows that for a given instant of emission, that is for a given value of a , $\sqrt{x^2+y^2}$ increases proportionately to the time. From the above table, I find the increase is '40 per hour, i.e. '35 in the interval between 3094 and 3095, and '38 in the interval 3095 to 3096. That the data are entirely consistent with this may be seen from the following:—

$$\text{Putting } \tau = \sqrt[4]{x^2+y^2} + \cdot 40(9^h 6^m - t)$$

we have

a .	τ .	Plate.
'061	'17	3096
'072	'40	3095
'104	'66	3096
'120	'79	3094
'122	'83	3095
'134	1'06	3096
'148	1'15	3095
'163	1'13	3094
'206	1'58	3094
'223	1'74	3095
'227	1'75	3096
'250	2'03	3095

showing a regular change of a with τ , irrespective of the particular photograph.

Again from equation (3)

$$40 = \sqrt{\frac{g}{2}},$$

whence

$$g = 0.32 \text{ per hour per hour} \\ = 0.80 \quad , \quad ,$$

and allowing for foreshortening the full solar repulsion

$$= 90,000 \text{ kilometres per hour per hour.}$$

This corresponds to $\mu = 4000$.

The following are the results of similar determinations on other nights :—

Reference No.	Date.	g .	μ .
1	Oct. 2	0'243 per hour per hour	730
2	"	0'310 " "	930
3	Oct. 14	3'80 " "	8,000
4	Oct. 15	5'20 " "	10,500
5	Oct. 27	7'20 " "	12,000
6	"	6'5 " "	11,000
7	"	9'8 " "	16,000
8	"	7' " "	12,000
9	Nov. 3	0'45 " "	700
10	Nov. 10	2'25 " "	3,500

Remarks.—1 and 2. These examples are noteworthy, because on this night, and this night only, there is substantial agreement between the values of the repulsion derived from the envelopes and from measures of points in the tail. The motion of the latter is exceptionally rapid on October 1 and 2, whereas this is nearly the lowest of the determinations from the envelopes. The envelopes are strongly hyperbolical. It may be noted (1) that the tail is filmy and resembles the envelopes in texture; (2) there are envelopes (dense, but with ill-defined edges) at a height of 5' from the nucleus towards the Sun—a height more than twice as great as on any other night.

3. A very good determination from four plates extending over $1\frac{3}{4}$ hours, the different points showing excellent agreement.

4. A rough determination.

5. Good.

6, 7, and 8. Much weaker.

9. Good determination, the results being very accordant.

10. Deduced solely from measures at *latus rectum* and *vertex*. Probably much less reliable than the other results.

§ 9. *Direct measurement of motion of matter projected towards the Sun.*—There is one isolated piece of evidence on this subject which

it seems well to record. On Oct. 18 (the night when there was the best example of the rapid formation of an envelope, see page 447) there is a rather well-defined portion of matter which has been projected towards the Sun. The measures give

Plate.	Time.		x .	y .	$\frac{dx}{dt}$.	$\frac{dy}{dt}$.
	h	m				
3173	7	58	+ '090	+ '244		
					'340	'028
3174	8	55	'416	'271		
					'339	'037
3175	9	50	'726	'305		

(Unit = 2'5.)

The hourly motion in x is thus 0'85.

Judging by the values of y , if $\frac{dy}{dt}$ is constant, the matter would appear to have left the nucleus at about 0^h; in that case, to satisfy the value of $\frac{dx}{dt}$, we must have $g = 0'22/\text{hr.}^2$. The evidence is rather uncertain; but as this would involve the matter reaching a height $x = -1'7$, g cannot very well be less. The corresponding value of μ is 420. This is greater (though not excessively greater) than the value of μ for the tail particles on that night.

§ 10. It is clear, then, that if we adopt the simple theory of the formation of envelopes, we are obliged to assume the action of repulsive forces 10 to 100 times as great as those which appear to act on the particles of the tail. Now there are two difficulties in the way of this. It is not clear that the envelopes can be sharply discriminated from the tail in the way this implies; as the envelope degenerates, it seems to mingle imperceptibly with the tail. Secondly, there are comets, such as Comet 1910 *a*, which seem to possess no tail, as distinct from the envelopes; probably among the comets which Bredichin examined were many of this type, and for these he must have actually measured the repulsive force on the envelopes and found it quite normal. Neither of these arguments is conclusive. It is quite possible that the envelope ultimately mingles with the tail, but any distinct point, whose motion can be directly measured, would almost necessarily be part of the tail proper, and not part of an envelope. Further, the envelopes of Comet Morehouse are apparently different in character from those to which Bredichin's method is applicable. Finally, there is considerable evidence that the solar repulsion ceases to act on the particles some time after their emission; this would account for the smaller values of μ obtained by those who study the remoter parts of the tail.

One assumption, however, is open to doubt, viz. that the velocity of projection is the same in all directions. The fact that the envelopes on the fore and rear sides of the comet do not

generally correspond, or only correspond imperfectly, seems to show that the cone of emission is restricted, or not of uniform force. This renders the second method of calculating g somewhat uncertain, but it does not affect the first method. A decrease of V as α increases would render the envelope more elliptic, and would thus counterbalance the more hyperbolic form which a smaller value of g would produce. At the same time it is hard to believe that the approximately parabolic form, which is generally observed, is the result of such a chance counterbalancing of two opposing tendencies.* I think it is likely that V may vary with α sufficiently to require an appreciable reduction of some of the values of g found by the second method; for this reason I place more stress on the less precise evidence of the first method; but even as regards the second method it will be found that any attempt to work out the problem on the assumption that the repulsion has the value found from the study of the tail particles leads at once to difficulties, for it requires a duration of the envelope for a time much longer than is actually observed.

If we admit the values of g found by the two methods that I have explained, the initial velocity V is found to be of the order 10 to 100 kms. per second. It is, I suppose, impossible that such velocities could be produced in gases simply escaping under physical pressure, and we must look to some form of electrical or radio-active action to account for it. In forming such a theory there are three facts to be borne in mind: (1) at any instant the initial velocities of all particles emitted in any direction must be very nearly the same (otherwise a sharp linear envelope could not be formed); (2) the velocity of emission must decrease rather rapidly with the time; (3) the conditions must admit of two or more envelopes in different stages of collapse existing simultaneously.

§ 11. *Attractive or Repulsive force to the Nucleus.*—Another important question is, What modification would result if the nucleus either attracts or repels the particles? It is not possible to work out generally the effect of such forces; but I have traced by quadratures the paths of the particles in special cases, and have concluded that such forces would not help to explain the rapid formation of the envelope. A fairly large attractive force to the nucleus (following the Newtonian law) would modify the envelope in the following ways:

- (1) It would make it more elliptical.
- (2) It would *increase* (slightly) the time taken for the envelope formation to spread from the vertex to the latus rectum.

* The assumption that V decreases as α increases was made by Bredichin and Bond to account for the deviation from the parabolic form in the envelopes of certain comets; but the former considered it more probable (seeing that the truly parabolic form was the usual one) that in these cases the emission took place with constant velocity, but was confined to a small cone instead of filling the whole hemisphere. This latter explanation, however, only applies when the envelope is a mere boundary, and not when it is a line of great density.

(3) It would throw the matter more into the part near the vertex; *i.e.* instead of the envelope between the Sun and the focal plane being formed by matter ejected in a cone of angle 45° , it would be formed by a much larger cone.

(4) The matter would tend to hang for some time near the envelope instead of dissipating rapidly.

I have not examined a repulsive force, owing to the difficulty of assuming reasonable initial conditions. Owing to the smallness of the nucleus compared with the envelope, it seems impossible (under the inverse square law) to devise initial conditions under which the repulsive force would have very much effect.

It seems therefore useless to look for a solution of the difficulty in this direction.

§ 12. *Evidence for the Fountain-Theory of Envelopes.*—I do not see any other modification of the fountain-theory which can possibly explain the rapid formation of the envelopes. It remains, therefore, either to accept the large repulsive forces or to give up the fountain-theory, at least in the case of the sharp-line envelopes of Comet Morehouse. I will therefore conclude by examining the evidence on which the theory rests. In the first place, we can in many cases actually see the fan-shaped fountain of matter pointing towards the Sun, which the theory demands. Secondly, whereas any feature of the tail tends to dissipate and become less well-marked in later photographs, the envelopes start by being indistinct, and gradually become denser and more sharply defined. This, of course, is a natural result of the contraction of an envelope on the fountain-theory, and it is hard to see how it could be brought about in any other way. If it were not for this we might be tempted to regard them as *isochrones* (although they are not the right shape), or some other appearance that *consists permanently of the same material*. This is especially tempting, as in the motion of an envelope, $\frac{dx}{dt}$ (y constant) seems to agree very well with $\frac{dx}{dt}$

for tail matter at the same distance from the nucleus. But material must almost necessarily dissipate with time, instead of increasing in density, as the envelopes do.

§ 13. Nichols and Hull's suggestion (*Ap. J.*, xvii. p. 354) that the envelope forms at a height where condensation (caused by expansion and cooling of the emitted vapours) takes place, is worth very careful attention, considering the difficulties involved in the fountain-theory. But why should condensation take place almost instantaneously over a paraboloidal surface? Why, in particular, should the condensation only take place at a much greater height from the nucleus in directions away from the Sun than in directions towards the Sun? Once condensed, the envelope-material would be subject to light pressure, and would probably tend to collect more condensing matter; the theory would account admirably for the observed motion, changes of form, and increase of density of the envelope. But the initial formation, nearly instantaneously and in a parabolic form, presents an apparently

insuperable difficulty.* The presence inside one another of a series of sharply-defined envelopes is also hard to understand.

§ 14. *Distribution of Density in the Paraboloid.*—The remarkable density and sharpness of the envelopes of Comet Morehouse is rather surprising when one has been accustomed to think of an envelope merely as a boundary. It is therefore desirable to show that it is a direct consequence of the fountain-theory, provided all the matter is expelled with the same velocity.

Take cylindrical coordinates (y, ϕ, x), x and y being measured as before. We have

$$\begin{aligned}x &= -Vt \cos \alpha + \frac{1}{2}gt^2 \\y &= Vt \sin \alpha \\ \dot{x} &= -V \cos \alpha + gt \\ \dot{y} &= V \sin \alpha.\end{aligned}$$

Whence $v^2 = \dot{x}^2 + \dot{y}^2 = V^2 + g^2t^2 - 2Vgt \cos \alpha$, where v is the velocity of the material; also $\dot{x} \sin \alpha + \dot{y} \cos \alpha = gt \sin \alpha$.

Now varying α and t ,

$$\begin{aligned}dx &= \dot{x}dt + Vt \sin \alpha d\alpha \\ dy &= \dot{y}dt + Vt \cos \alpha d\alpha.\end{aligned}$$

If we make dx, dy perpendicular to the velocity of the particles and in the plane xy , we must have

$$\dot{x}dx + \dot{y}dy = 0.$$

This leads to

$$(V^2 + g^2t^2 - 2Vgt \cos \alpha)dt = -Vgt^2 \sin \alpha d\alpha \quad . \quad (4)$$

Now $ds^2 = dx^2 + dy^2$

$$= (\dot{x}^2 + \dot{y}^2)dt^2 + V^2t^2d\alpha^2 + 2Vt(\dot{x} \sin \alpha + \dot{y} \cos \alpha)dt d\alpha.$$

Therefore, using condition (4) for an element perpendicular to the direction of motion of the material, we find

$$ds^2 = \frac{V^2t^2d\alpha^2}{(V^2 + g^2t^2 - 2Vgt \cos \alpha)} (V - gt \cos \alpha)^2.$$

Suppose that a steady state is reached and matter is emitted uniformly in all directions. Consider the matter emitted in an element of solid angle $\sin \alpha d\alpha d\phi$. It will flow in a tube of cross

* It is possible that a swarm of ions proceeding from the Sun and encountering the comet would, if repelled by the nucleus, part to either side; their paths would have as an inner envelope a roughly parabolic curve. The ions would serve as nuclei for condensation of the expelled gases.

section $ds.yd\phi$, and the quantity flowing across any cross section in unit time, namely,

$$\rho v ds.yd\phi,$$

will be proportional to $\sin adad\phi$.

Therefore $\rho V t da (V \sim gt \cos a). V t \sin ad\phi \propto \sin adad\phi$.

Or the density ρ varies inversely as

$$t^2(V \sim gt \cos a).$$

On the envelope $V = gt \cos a$ and the density is infinite. As there are two tubes of flow passing any given point, corresponding to different solutions of equation (1), the total density is the sum of two parts, each given by the above expression.

Putting $u = \frac{1}{2}gt^2$, by a slight transformation the density may be written

$$\rho \propto \frac{1}{u_1 |(2a+x-u_1)|} + \frac{1}{u_2 |(2a+x-u_2)|} \quad (5)$$

where u_1, u_2 are roots of

$$u^2 - 2(x+2a)u + x^2 + y^2 = 0.$$

In most cases, if not in all, the envelope-forming matter is only projected in the hemisphere towards the Sun. In that case, within the parabola K O K' (see diagram, p. 444), formed by displacing the envelope a distance a along Ox, one of the tubes of flowing material will be missing, and ρ for such points consists of a single term. On this supposition I give the relative density of material in the paraboloid at different points in the cross sections $x=0$ and $x=a$, that is, cross sections drawn through O and M respectively. η is the ordinate of the envelope for the cross section considered.

Distribution of Matter in the Paraboloid.

$\begin{smallmatrix} y \\ x \end{smallmatrix} \eta$	0.	.2.	.4.	.6.	.707.	.8.	.9.	.95.	.99.	1.00.
0	∞	136.0	34.1	17.4	14.1	13.0	14.1	17.8	36.2	∞
a	0.61	0.62	0.69	0.84	6.00	5.77	6.53	8.27	17.2	∞

This table shows that the increase of density at the envelope is very sudden, so that a sharply-defined line may be expected.

There is a minimum density at about $\frac{y}{\eta} = .8$, *i.e.* just within the

envelope—a feature which is often noticeable on the photographs. Taking the cross section $x=a$, we see that for $y/\eta < .707$ there is very little material (owing to no ejection taking place in the hemisphere away from the Sun). This feature is very conspicuous in comets such as 1910 *a*, which show a divided “tail” with a

dark space along the radius vector. There is no need to assume (as has been sometimes suggested) that the nucleus causes a sort of shadow; the figures show that the amount of matter projected towards the Sun and falling back into this space is in any case very insignificant. For $\frac{y}{\eta} > \cdot 707$ both the theoretical paths

correspond to matter projected towards the Sun, so that there are now two streams contributing to the density. As y increases, the density diminishes at first, and then increases as the envelope is reached. There is thus a line of minimum density running within and near the envelope. The broken line UV represents (roughly) part of its course. This seems to explain Mr. Hinks' observation in the case of Comet 1910 *a*, that in each of the two symmetrical streams of envelope-matter the density was greatest at the edges.

It must be borne in mind that the appearance of the paraboloid, whose density is given above, will differ somewhat from the appearance of a plane section of it. It is not worth while working out the result mathematically, as the appearance must depend on the inclination of the axis to the line of vision, but the general considerations of the above paragraph will evidently hold good for the three-dimensional problem; and I think that the phenomenon of a minimum density occurring just within the envelope will be more prominent than in the two-dimensional case.

§ 15. *Conclusion.* — I feel personally reluctant to accept the very large repulsive forces indicated by the investigations of this paper, which seem to differentiate the envelope-particles from the tail-particles; on the other hand, I am not able to advocate any alternative explanation of the facts. The question clearly needs further examination, and I hope that the study of Halley's Comet may throw light on the matter. But it is necessary to recognise that Comet Morehouse was very abnormal in many respects, and it may be some time before a comet appears showing strictly similar phenomena.

Royal Observatory, Greenwich:
1910 March 10.

On the Spectrum of the Daylight Comet 1910 a.

By H. F. Newall, M.A., F.R.S.

The daylight comet was examined spectroscopically at the Cambridge Observatory, and exhibited interesting phenomena, which deserve to be recorded; but in describing them I have to adopt origins for the monochromatic radiations which are not based on measurements of the spectrum, for the comet eluded all our efforts to get a photograph of its spectrum. Our ill success may be attributed partly to the low altitude of the comet, and mainly to the dewing of the object-glass of the 25-inch equatorial. At the end of January and early in February, when we have frosty nights and damp, muggy mornings, the dewing has often been troublesome; and though, after the failure of various heating operations, which have on previous occasions proved successful, the object-glass was dismounted in order that the dew might be wiped off the inner surface, yet even after this process was carried out on the 27th January there were recorded with the two-prism spectrograph only the faintest traces of a continuous spectrum. In what follows, I have therefore to base the identification of the brilliant yellow monochromatic radiation with sodium as origin on the telegraphic announcement from the Lick Observatory.

The spectroscopic observations which I wish to record were made with a direct-vision prism inserted between the eye and the eyepiece of the telescope; sometimes the 25-inch equatorial was used, sometimes the 6½-inch finder attached to it.

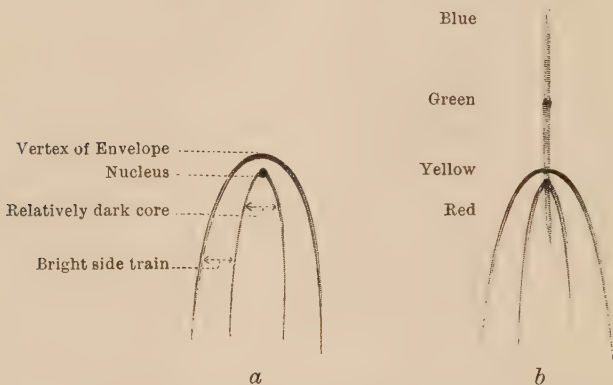
On Saturday, January 22, it appeared that the main part of the luminosity of the envelope and side trains were due to sodium. Fig. *a* represents the telescopic appearance of the head of the comet, showing a brilliant nucleus with a coma and distinct envelope trailing backwards in two side trains, leaving a relatively dark core between them behind the nucleus.

When the prism was held so that its principal plane was parallel with the axis of the comet (fig. *b*), the marked features were—

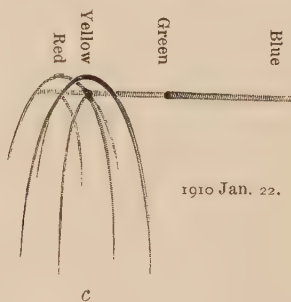
1. Intensely strong continuous spectrum of the nucleus with two distinct bright beads corresponding with the nucleus, one in the yellow (sodium), the other, less bright, in the green (?magnesium).

2. The envelope and side trains gave very feeble continuous spectrum, but a very strong image in monochromatic yellow (sodium); the outer edge of the envelope was brightest, and the dark core was seen to show some orange and reddish light on either side of the bright red continuous spectrum of the nucleus. The origin of this red and orange light was not clear till the prism was turned through a right angle. It was then seen (fig. *c*) that there were several monochromatic images of the vertex of the envelope in addition to that in sodium light. Three images were seen with certainty in the orange and red, and at times a fourth was suspected.

But there were no signs of green or blue images of the vertex of the envelope; the continuous spectrum of the nucleus formed a straight strip, which was clearly *below* the tangents to the red, orange and yellow images of the vertex of the envelope. The green spot, provisionally attributed to magnesium in the absence of signs of hydrocarbon bands, was seen only as if confined to the nucleus.



The sodium was traceable in the side trains to a distance of $7\frac{3}{4}'$ (minutes of arc) behind the nucleus; the orange and red images extended only about half as far. These distances were estimated by reference to a réseau of wires in the focal plane of the $6\frac{1}{2}$ -inch finder. When first the estimate was made, I thought I could trace the side trains about $12'$ behind the nucleus, but I found that a



great part of this was due to the superposition of the orange and red images on the sodium image; and the extension $7\frac{3}{4}'$ was seen with the prism set as in fig. *c*.

It must be remembered that the image of the comet was seen magnified 45-fold by the telescope, and the image was examined with a direct-vision prism, which gave a dispersion of $45'$ between the lithium and sodium lines, $75'$ from the yellow to the end of the green, and $90'$ from the end of the green to the limit of visibility in the violet. The figures *a*, *b*, *c* give the approximate appearance

as seen in the telescope with the direct-vision prism. The extension of the sodium, being say 8', appeared in the telescope under an angle $\frac{8 \times 45}{60}$ degrees = 6°, and then it seemed about eight times as long as the red end of the spectrum of the nucleus beyond the sodium line.

On January 26 the form of the head of the comet had slightly changed, in the sense of its appearing to have a less well-defined envelope. The prism showed that the spectrum of the nucleus exhibited the hydrocarbon bands, red, yellow, green, and blue, quite well marked, on a faint continuous spectrum.



There were also two monochromatic images of the nucleus, each with two line-like trails, coinciding with the inner edges of the side trains (fig. *e*), the one pair yellow and attributed to sodium, the other orange and much fainter, in fact so much so that it was not seen till the prism was turned to give dispersion *across* the axis of the comet (fig. *f*).

The extension of the sodium arrowhead was rather more than 1', that of the orange arrowhead rather less. The vertex of the envelope was not seen in these colours.

On January 28 sodium was seen only in the nucleus, very much enfeebled. The hydrocarbon bands were not nearly so distinct.

The progressive change from January 22 to January 26 and to January 28 is striking. It suggests that on January 22 the sodium radiation was evoked in the coma up to the edge of the envelope in front of the nucleus, and then carried back into the side trains behind the nucleus; on January 26 the sodium was evoked only close to the nucleus, and then at once carried back behind the nucleus; whilst on January 28 the sodium radiation was confined to the nucleus.

Observations of Comet 1910 a, made at the Cambridge Observatory.
By Arthur R. Hinks, M.A. (Plates 12, 13.)

The following notes refer to the physical appearance of the comet. My observations of position have been published in *A.N.* 4392.

1910. *Jan.* 19.—Swept for the comet in a hazy sky from 2^h G.M.T., with Sheepshanks (12-in.) and Adams (3½-in.) equatorials. Shortly before 4^h it was picked up by Mr. J. B. Hubrecht with 4-in., and a glance at his instrument showed that comet was about 3° N. of Sun. This enabled me to find it immediately with the Sheepshanks. Nucleus estimated brighter than Mercury. The few available minutes used for circle settings for position. Naked eye, tail 2°.

Jan. 20.—Observations interrupted by clouds. Comet found in Sheepshanks finder, G.M.T. 4^h 22^m. Circle readings for position. Two plates unsuccessful.

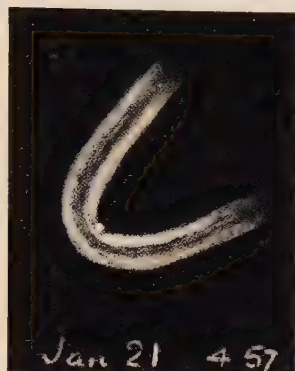
Note.—The Sheepshanks finder is a large eyepiece with field about 70' and power 20. With this low power the diameter of the emergent beam is about 0.4 in., and consequently only the central 6 inches of the objective are utilised. With the focal length 19.3 feet, this gives a ratio nearly 40:1; hence the images of extended surfaces are unduly faint in this eyepiece.

Jan. 21.—After observations for position, the head examined with power 180, G.M.T. 4^h 57^m. Nucleus intense and very stellar on the extreme inner edge of the inner parabolic arc. The two parabolic arcs enveloping the nucleus separated by a decidedly darker space (see Plate 12 (1)).

Jan. 22.—G.M.T. 5^h 3^m. Power 180. The dark space less pronounced than on previous night, but seeing much worse. (Plate 12 (2)). With a direct-vision prism held over the eyepiece, the base of the tail seen strongly in yellow; extent about the same with and without prism. No trace of tail except in the yellow line. Nucleus gave a continuous spectrum with decided bright line in blue-green and one suspected in the red. Looked carefully to see whether nucleus showed yellow bright line. Judged that at point where monochromatic image of tail covered continuous spectrum of nucleus, the intensity not sensibly enhanced. Hence yellow not conspicuously bright in nucleus spectrum.

[This observation is contradicted by other observers of greater experience, and should therefore be received with caution. But as it was made deliberately, I give it for what it may be worth.]

Jan. 25.—Power 180. Nucleus much less bright than on January 22. Head not so well developed. Only traces of dark structure visible (Plate 12 (3)). Spectrum much less brilliant. Could not see yellow tail except as a patch on the continuous spectrum of nucleus. Bright line in blue-green stronger. No other features certain.



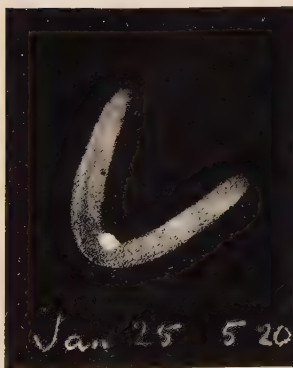
Jan 21 4 57

(1) POWER 180.



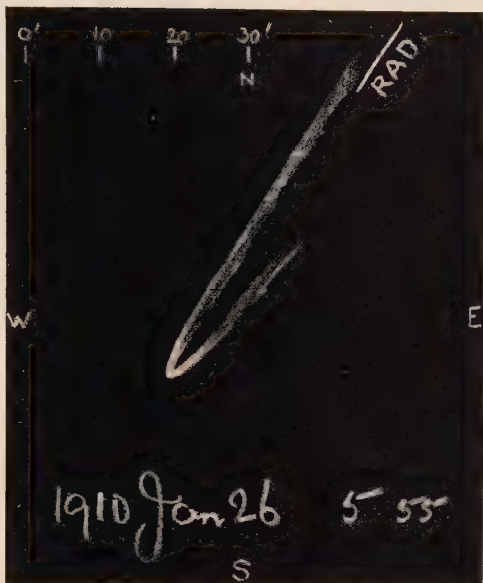
Jan 22 5 3

(2) POWER 180.

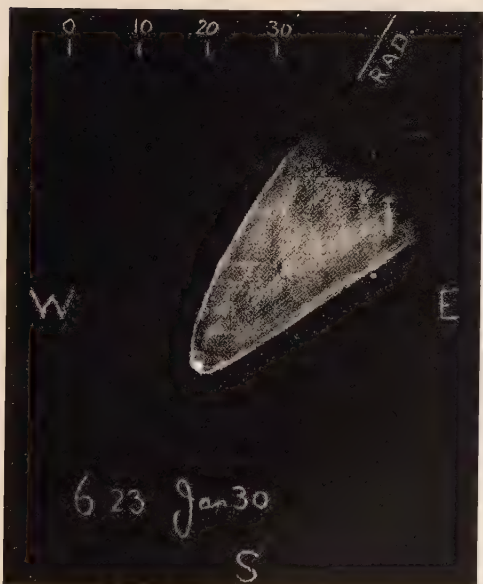


Jan 25 5 20

(3) POWER 180.



(4) POWER 20.



(5) POWER 20.

COMET 1910a.

DRAWN AT CAMBRIDGE.—A. R. HINKS.



(1) 1910 JAN. 30. 6 H. 45 M. NAKED EYE.

(2) COMET'S TAIL AND ZODIACAL LIGHT. NAKED EYE.
1910 JAN. 30. 6 H. 55 M.

With finding eyepiece the two branches of tail about equal. Sketch controlled by the reticule in eyepiece. Total breadth of tail 6' at 20' from nucleus. G.M.T. 5^h 3^m.

With naked eye, at 6^h 5^m, tail about 10°, curved to south at tip.

Jan. 26.—Power 180. With direct-vision prism, spectrum decidedly banded in green. Yellow line nearly as prominent as previous night, and traces of the envelopes visible in the yellow [this observation doubtful]. Dark band suspected on red side of yellow line; perhaps contrast.

With finding eyepiece, the southern branch of tail decidedly less extended than the northern [Plate 12 (4)]. As the tail was nearly vertical, this must be a real difference. Total breadth of tail 7½' at 20' from nucleus. G.M.T. 5^h 55^m.

Note.—In the drawings made with the finding eyepiece [Plate 12 (4) and (5)] the field is reversed N. and S., as shown, because the instrument is a *coudé*.

With naked eye, at 6^h 10^m, tail about 18°. Sketched with surrounding stars. Axis of tail near nucleus makes angle 18° with direction of radius from Sun, as laid down by means of stars on sketch. Tail sharp on northern edge, curving to south, and diffuse on southern edge.

Jan. 28.—Observations for position only, through clouds.

Jan. 30.—Sky very clear.

Power 180. Nucleus considerably fainter, and no structure visible about it. Spectrum changed. Yellow line invisible. Bright patch in green.

With finding eyepiece, at 5^h 57^m, tail hardly visible. At 6^h 23^m, careful drawing controlled by reticule showed tail markedly broader and unsymmetrical. At 20' distance from nucleus, breadth of tail 18'. Bifurcation not noticeable. Filled with continuous faint light. A star (*α* Aquarii) within 1' of southern edge [Plate 12 (5)].

With naked eye, at 6^h 30^m, tail about 20°. Sketched with surrounding stars. Axis of principal tail near nucleus makes an angle 23° with direction of radius from Sun, as laid down by means of stars on sketch. Short subsidiary tail to south makes angle about 40°. Another sketch at 6^h 30^m shows tail making angle 20° with radius, and subsidiary tail 33°. Length of tail about 30°. Curved sharply to south just above *α* Pegasi, and apparently continued in direction of dotted line [Plate 13 (1)]. Scale value of plate determined by stars, which, as drawn, agree fairly well with true relative distances. Zodiacal light becoming well seen to south. The stars lettered are *α*, *β*, *ε* Pegasi, and *α* Aquarii. The object marked V is Venus.

As the night became darker, the zodiacal light was strong, its apex 2° N.W. of Saturn. The comet's head was lost in haze, but the tail, about 50° long, curved above *α* Pegasi and below *γ*, till it exactly joined the apex of the zodiacal light. The two were of about the same intensity near their junction. A careful drawing with the stars was made at 6^h 55^m [Plate 13 (2)]. The scale value and the position of the ecliptic were determined next morning

from the stars as drawn. The ecliptic thus determined and the latitude of Saturn 2° S. confirm the general accuracy of the outlines of the zodiacal light and the position of its apex. The stars lettered are α , γ , θ Pegasi.

Feb. 3.—Cloud and haze. With power 180, the head an ill-defined nebulous patch, without stellar nucleus.

Feb. 4.—Very dense haze low down. At $6^{\text{h}} 20^{\text{m}}$, comet scarcely visible. Could see no trace of tail. At $6^{\text{h}} 40^{\text{m}}$, zodiacal light strongly visible, but no tail.

Feb. 6.—Comet picked up at 6^{h} , and lost immediately in clouds. This was the last observation of the comet at Cambridge.

The drawings were made with white chalk on black paper. I have thought it better to reproduce the rough sketches as they were made rather than to attempt finished drawings. Our experience with Comet 1910 *a* suggests that there may still be occasions on which drawing may legitimately supplement photography; and it may be well not to lose sight of this fact when preparing for the observation of Halley's Comet in April and May.

The directions of the radius from Sun in the finder drawings are computed; in the naked-eye drawings they are taken from a large globe.

Cambridge Observatory:
1910 March 10.

Notes on Comet 1910 a. By the Rev. W. Sidgreaves, S.J., and the Rev. A. L. Cortie, S.J.

The following observations of the physical appearance of the comet were made at this observatory.

1. On January 25 it was observed at 5 45 p.m., and for about a quarter of an hour afterwards, until hidden by clouds. With a low-power eyepiece in the 15-inch equatorial, the head was compact, and the nucleus brighter than a first magnitude star. There was a wide, dark segment dividing the two branches of the tail. To the naked eye the tail was slightly curved away from the Sun, and was about 5° long.

2. On January 26 it was seen in a perfectly clear sky from 5.40 to 7.0 p.m. In the 15-inch equatorial the nucleus was quite stellar in appearance, and the dark segment between the two branches of the tail was broad, the inclination of the two branches of the tail to one another, as measured on a photograph, being about 10° . At an angle of about 40° to the axis of the tail, on the eastern side, was a fainter luminous cloud of particles, possibly part of an outer envelope. But on the western side the tail was sharp, and slightly convex to the Sun. The whole appearance of the tail was very similar to that of Donati's Comet as drawn by Bond and Pape, except that the western branch was more diffuse on its following side. Two plates were taken with a 6-inch Dallmeyer portrait

lens, on an Imperial isochromatic plate, from 6.20 to 6.30 p.m., and on an Imperial flashlight plate from 6.32 to 6.44. Both show the appearances detailed above very clearly, and the tail extends to the edge of both plates, or for nearly 5° in length. The breadth of the tail on the plate, near the nucleus, is $28'$, and at a distance of $4^\circ 57'$ from the nucleus is $36'$. The outer fainter cloud of particles or envelope extends, as photographed, $57'$ from the nucleus, and had it been photographed to the edge of the plate would have subtended $1^\circ 19'$, which agrees fairly well with an estimated naked-eye length of tail of 10° , and a breadth at the extremity of 3° . The measures on the plates are approximate, having been made with an ordinary inch scale. To the naked eye the nucleus was as bright as a first-magnitude star, and the tail rose almost vertically from the horizon, with a slight convexity towards the position of the Sun. It persisted after the head had set until the moonlight blotted it out. An excellent drawing of the comet was made, with the aid of a small pocket telescope, $1\frac{1}{2}$ inch aperture and 17 inches focal length, by Mr. William McKeon, an assistant at the observatory. This shows all the details of the photographs, even the luminous wing. By reference to α Equulei, the tail as seen in this small instrument extended 7° . The nucleus appeared to be at the extremity of the tail, without any encircling envelopes. But in the 15-inch, the envelopes were clearly defined, as observed by Mr. D. Hutchinson. The spectrum of the nucleus was observed for a few minutes with a small direct-vision McClean spectroscope attached to the 15-inch equatorial. The spectrum appeared to be continuous, with a decided brightening in the green, presumably due to a hydrocarbon band. The yellow sodium line was not seen. But the comet was, to the naked eye, decidedly of a yellowish tinge, possibly due to its being so near to the horizon.

3. On January 29 an image of the comet was obtained on an Imperial flashlight plate, which showed an impression of the tail, after an exposure of 70 minutes, from 5.30 to 6.40 p.m., extending $1^\circ 36'$ in length. The comet had diminished in brilliancy, and the head was more diffuse than on January 26. To the naked eye, by reference to the star α Pegasi, the tail extended over 27° .

4. On February 4, the nucleus, in the 15-inch equatorial, was no longer a shapely defined stellar object, but was very much more diffuse than on former occasions of observation. An exposure was made from 6.46 to 7.2 p.m., but a very long development only brought out a faint image of the nucleus on the plate. It was still a fine naked-eye object, the tail extending to λ Pegasi, being $22\frac{1}{2}^\circ$ in length. On the western side of the tail, and at its upper extremity, was an extensive luminous haze, as if a cloud of particles was following in the wake of the tail. This would give an appearance of a bending of the tail at its upper extremity.

*Measures of Double Stars made at the Royal Observatory,
Edinburgh.*

(Communicated by the Astronomer Royal for Scotland.)

The following observations of double stars were made with the 15-inch refractor during the year 1909 by Mr. Storey. As may be seen from the last column, most of the observations were made in the latter half of the year. A Working Catalogue of stars within 40° of the Pole was kindly compiled by Mr. Lewis, as it seemed desirable in this latitude to observe stars not so favourably situated for the more Southern observatories.

Name of Star.	R. A. 1910.	Decl. 1910.	Position Angle.	Dist.	No. of Nights.	Mags.	Epoch 1909+.
	h m						
Krueger 67	... 0 1	60 22	163° 1	3" 08	2	9' 0, 9' 2	'879
Σ 3062	... 0 1	57 56	359° 6	1' 68	1	6' 9, 8' 0	'783
Espin 114	... 0 4	66 39	170° 2	4' 24	1	8' 7, 11' 2	'969
β 253	... 0 6	58 1	40° 3	0' 59	1	8' 3, 8' 5	'972
Aitken 801	... 0 6	75 1	237° 8	2' 15	1	9' 1, 9' 5	'934
Krueger 1	... 0 6	57 20	188° 1	2' 10	1	9' 2, 9' 5	'849
Σ 7	... 0 7	55 28	219° 1	1' 30	1	8' 0, 8' 5	'980
O Σ 1	... 0 7	65 38	201° 9	1' 54	1	7' 2, 9' 9	'980
Krueger 3	... 0 10	55 12	43° 8	3' 68	2	9 9	'879
Σ 13	... 0 11	76 27	85° 5	0' 89	2	6' 6, 7' 1	'976
Aitken 904	... 0 12	57 6	0° 9	3' 53	1	8' 9, 10' 0	'969
Σ 18	... 0 12	67 8	87° 5	0' 81	1	8' 2, 8' 8	'972
Krueger 4	... 0 14	59 12	188° 9	2' 24	1	8' 5, 9' 0	'824
Hussey 1006	... 0 16	65 18	197° 2	3' 08	1	9' 0, 9' 4	'824
Holden 2	... 0 20	51 4	335° 7	2' 84	1	8' 8, 8' 8	'857
Krueger 7	... 0 25	59 12	8° 7	4' 03	1	9' 5, 9' 7	'969
β 107	... 0 26	62 51	352° 3	6' 13	2	8' 0, 9' 6	'984
Σ 34	... 0 26	77 37	337° 1	6' 28	1	8' 7, 8' 8	'969
Σ 43	... 0 31	60 1	167° 7	5' 07	1	8' 5, 9' 0	'936
Holden 3	... 0 40	53 4	53° 7	2' 85	2	8' 5, 8' 6	'864
Σ 59	... 0 42	50 57	146° 4	2' 22	1	7' 2, 8' 1	'857
Σ 60	... 0 43	57 20	244° 4	5' 93	1	4' 0, 7' 6	'969
β 781	... 0 45	68 30	26° 6	0' 99	2	8' 1, 8' 6	'848
Σ 65	... 0 47	68 23	34° 2	3' 16	2	8' 0, 8' 0	'826
Espin 118	... 0 49	63 53	244° 3	2' 59	1	8' 6, 8' 7	'857
Holden 4	... 0 52	53 55	123° 4	0' 95	1	8' 5, 9' 0	'980

Name of Star.	R.A. 1910.	Decl. 1910.	Position Angle.	Dist.	No. of Nights.	Mags.	Epoch 1909+.
	h m	° '	°	"			
Krueger 10	... 1 6	60 43	277°6	3'17	1	9'5, 10'5	'999
Σ 96	... 1 6	64 32	285°6	1'11	2	7'8, 8'8	'870
β 258	... 1 7	61 14	253°1	0'94	1	6'2, 9'0	'928
Hussey 1023	... 1 7	65 8	2°6	3'63	1	8'7, 10'2	'969
Hussey 1025	... 1 10	68 12	199°1	2'94	1	9'0, 9'8	'972
OΣ 28	... 1 10	80 23	309°4	0'87	1	7'0, 8'5	'928
Krueger 11	... 1 13	61 7	240°2	1'87	1	9'3, 9'3	'980
β 782	... 1 15	55 45	78°5	2'84	1	8'0, 9'6	'999
Σ 114	... 1 17	72 22	353°8	3'51	1	7'2, 10'4	'999
Σ 115	... 1 17	57 40	149°3	1'12	1	7'3, 7'5	'972
Σ 130	... 1 28	69 26	186°1	7'53	1	8'0, 9'0	'030
Herschel 1085	... 1 32	63 15	117°6	3'87	2	9'5, 9'5	'950
Σ 169	... 1 46	69 36	137°0	5'49	1	8'5, 11'0	'030
Σ 170	... 1 47	75 47	248°8	3'27	2	6'7, 7'5	'466
Σ 182	... 1 50	60 51	301°9	4'02	2	7'0, 7'0	'466
Σ 185	... 1 53	75 4	22°2	1'24	1	7'0, 8'5	'980
Σ 191	... 1 55	73 25	190°0	5'63	1	6'2, 8'5	'049
Krueger 13	... 1 58	56 35	346°4	4'15	2	9'2, 9'5	'509
Σ 235	... 2 10	55 29	46°7	1'97	1	8'5, 9'0	'925
β 786	... 2 11	55 20	352°0	4'97	1	8'5, 9'9	'999
Σ 233	... 2 12	75 58	276°7	2'19	2	8'5, 9'0	'492
Σ 262 AB	... 2 21	67 0	253°3	2'38	3	4'2, 7'1	'330
AC	...		109°8	7'68	3	4'2, 8'1	'330
Σ 268	... 2 23	55 8	127°4	2'74	1	6'9, 8'2	'980
Σ 272	... 2 26	58 4	37°2	1'91	2	8'2, 8'2	'073
Σ 282	... 2 33	65 16	111°7	7'42	1	8'3, 8'3	'071
Σ 283	... 2 33	61 6	205°2	1 83	2	8'0, 8'8	'515
Σ 314	... 2 46	52 38	302°9	1'31	1	6'9, 7'1	'980
Σ 312	... 2 47	72 32	20°4	3'06	2	7'1, 8'0	'056
Σ 317	... 2 49	68 50	84°4	3'84	1	7'8, 9'5	'019
Σ 320	... 2 53	79 4	228°7	4'73	2	6'3, 9'5	'034
Krueger 17	... 2 56	60 29	222°6	3'56	2	9'0, 9'1	'947
Herschel 1130	... 2 58	67 20	207°9	7'73	1	10 11	'052
Krueger 18	... 2 58	57 23	277°6	1'36	1	9'2, 9'3	'980
OΣ 50	... 3 3	71 13	196°7	1'48	2	7'5, 7'5	'548
Σ 368	... 3 13	68 11	165°9	2'26	1	8'5, 8'5	'096
Σ 384	... 3 21	59 35	278°0	2'03	1	7'8, 9'0	'052

Name of Star.	R.A. 1910.	Decl. 1910.	Position Angle.	Dist.	No. of Nights.	Mags.	Epoch 1909+.
	h m	° '	°	"			
Σ 386 ...	3 21	54 51	60°6	2'48	1	8·8, 8·8	'980
Σ 385 ...	3 21	59 37	161°9	2'30	1	4·7, 9·0	'980
Σ 419 ...	3 33	69 33	76°6	3'42	1	7·2, 7·2	'019
Hussey 1063 ...	3 34	62 39	336°2	2'92	1	9·0, 9·5	'925
A.G. 71 ...	3 35	65 49	240°8	8'04	1	9·4, 9·5	'049
Σ 445 ...	3 43	59 51	258°5	3'13	1	8·2, 9·2	'925
Bird 1 AB	3 54	62 15	227°9	1'87	2	7·2, 8·5	'508
AC			175°4	9'11	2	7·2, 9·5	'508
Espin 122 ...	3 56	61 55	247°8	5'29	1	8·6, 10·5	'049
Σ 472 ..	3 56	71 47	14°9	7'19	1	9·2, 9·7	'052
Σ 503 ...	4 8	63 56	226°9	4'68	1	8·8, 8·8	'019
Σ 504 ...	4 9	67 21	266°7	7'31	1	8·5, 10·0	'118
Σ 506 ...	4 10	70 8	292°7	10'05	1	9·0, 9·2	'071
Krueger 22 ...	4 10	60 37	188°3	3'48	1	9·3, 9·4	'052
Σ 513 ...	4 11	61 22	56°8	5'67	1	7·8, 9·7	'041
Σ 538 ...	4 21	64 1	217°6	7'38	1	8·5, 9·7	'049
Krueger 23 ...	4 23	55 18	131°7	4'37	1	9·0, 9·3	'999
Σ 553 ...	4 26	50 52	135°0	3'36	1	8·0, 8·5	'999
Herschel 1148 ...	4 31	68 18	127°1	7'97	1	9·6, 10·0	'118
Espin 56 ...	4 35	58 34	204°8	9'66	2	8·5, 8·8	'115
Σ 604 ...	4 49	69 55	39°7	2'44	1	8·1, 8·9	'019
A.G. 84 ...	4 55	54 42	160°4	3'90	1	8·8, 9·2	'999
Σ 558 ...	4 56	86 47	206°4	3'21	1	8·4, 9·9	'052
Σ 625 ...	4 56	58 44	112°0	4'99	1	8·2, 9·8	'041
Σ 638 ...	5 4	69 44	221°0	5'48	2	7·5, 8·5	'030
Σ 659 ...	5 13	64 49	312°2	5'83	2	8·7, 9·7	'096
Σ 677 ...	5 16	63 18	248°1	1'49	1	7·7, 8·0	'049
Σ 720 ...	5 28	63 27	167°2	6'40	2	8·2, 9·3	'045
Σ 780 AB	5 41	65 44	106°5	3'87	2	6·7, 7·9	'095
AC			148°2	11'84	1	6·7, 10·2	'118
Σ 868 ...	6 12	73 57	39°9	3'67	2	8·5, 9·0	'074
Σ 916 ...	6 26	56 43	255°1	9'72	2	8·5, 9·8	'093
Σ 946 ...	6 36	59 33	130°8	4'33	2	7·2, 9·0	'114
Σ 948 AB	6 38	59 32	116°2	1'76	1	5·2, 6·1	'052
AC			305°5	8'79	1	5·2, 7·4	'052
Σ 958 ...	6 40	55 48	258°1	5'36	1	6·0, 6·0	'118
Krueger 29 ...	6 53	56 59	357°5	6'54	1	9·0, 9·1	'099
Σ 1039 ...	7 10	63 41	206°7	3'21	1	8·8, 9·5	'099
Σ 1127 AB	7 38	64 19	339°3	5'53	2	6·2, 8·0	'117

Name of Star.	R.A. 1910. h m	Decl. 1910. °	Position Angle. °	Dist. "	No. of Nights.	Mags.	Epoch 1909+.
AC	...		177°2	11°80	2	6·2, 9·2	·117
Σ 1280 ...	8 47	70 59	45°6	4°71	1	7·5, 7·6	·216
Σ 1359 ...	9 26	56 39	63°5	7°76	1	8·5, 9·2	·216
Σ 1362 ...	9 29	73 29	132°9	5°34	1	7°0, 7°0	·211
Σ 1373 ...	9 36	77 8	128°7	1°99	1	8·2, 9·5	·216
Σ 1386 ...	9 47	69 20	292°7	1°86	1	8·2, 8·2	·257
Σ 1479 ...	10 53	83 43	19°7	4°90	2	8°0, 9°0	·337
OΣ 233 ...	11 11	67 10	340°4	4°93	1	6·9, 9·8	·298
Krueger 38 ...	11 27	60 34	50°4	2°68	2	9°0, 9·2	·338
Σ 1553 ...	11 30	56 38	169°5	5°95	2	7·3, 7·8	·281
Σ 1559 ...	11 32	64 51	326°8	1°95	2	6·7, 7·7	·339
Krueger 40 ...	11 41	60 20	277°8	3°38	2	9·2, 9·5	·343
Σ 1630 ...	12 13	56 51	168°8	2°60	2	8·3, 9°0	·362
Σ 1654 ...	12 27	75 18	25°4	3°99	2	7·3, 8·8	·334
Σ 1717 ...	12 45	89 10	332°8	8°69	1	8·6, 10°0	·342
Krueger 41 ...	12 58	56 54	338°2	3°85	2	9°0, 9·4	·363
Σ 1720 ...	12 58	83 25	335°4	1°53	2	8·4, 8·7	·365
Σ 1767 ...	13 31	68 13	350°2	4°80	2	8°0, 8·5	·370
Krueger 42 ...	13 32	60 23	215°9	3°96	2	9·5, 9·6	·362
Σ 1771 ...	13 34	70 14	75°6	1°73	2	7·8, 8·5	·374
Σ 1820 ...	14 10	55 45	80°4	2°19	2	8·2, 8·5	·400
Σ 1830 AB	14 12	57 5	294°5	7°16	2	8·5, 9·8	·400
AC	...		80°1	35°86	2	8·5, 12°1	·400
Σ 1831 ...	14 12	57 8	139°4	6°48	2	6·3, 9°0	·400
Σ 1878 ...	14 39	61 38	329°3	3°66	2	7°0, 9·2	·423
Σ 1892 ...	14 50	59 26	242°7	3°12	2	8·5, 9·7	·423
Σ 3125 ...	15 24	67 23	272°1	2°13	2	8·7, 9°0	·441
OΣ 299 ...	15 32	64 12	23°0	3°59	2	7·2, 9·5	·441
Σ 2036 ...	16 11	72 48	232°9	2°24	1	8·8, 10°3	·444
β 41 ...	16 17	61 40	60°5	2°33	1	9°0, 10°7	·444
Σ 2045 ...	16 18	61 43	181°8	2°47	1	8°0, 9·2	·444
Σ 2130 ...	17 3	54 36	132°9	2°43	2	5°0, 5°1	·745
Krueger 46 ...	17 18	58 37	64°1	1°74	1	8·8, 9°0	·597
β 1201 ...	17 26	67 51	338°3	0°53	2	7·8, 7·8	·709
Σ 2199 ...	17 37	55 48	80°6	1°53	1	7·2, 7·8	·693
Σ 2207 ...	17 37	67 10	125°9	0°90	2	8°0, 8·5	·652
Σ 2218 ...	17 39	63 43	344°2	2°20	2	6·5, 7·7	·649
Σ 2284 ...	18 1	65 57	192°5	3°78	1	7·6, 9·2	·605
Σ 2278 AB	18 1	56 26	25°1	37°66	1	6·8, 7·3	·780

Name of Star.	R.A. 1910.	Decl. 1910.	Position Angle.	Dist.	No. of Nights.	Mags.	Epoch 1909+.
	h m	°	°	"			
BC ...			146°3	6'37	2	6·8, 7·8	'782
Hussey 926 ...	18 9	64 13	268°0	4'50	1	9·1, 9·3	'624
Σ 2307 ...	18 12	69 13	206°4	4'57	1	8·5, 8·5	'605
Hussey 65 ...	18 17	84 35	267°1	1'43	1	9·2, 10·0	'693
OΣ 353 ...	18 22	71 17	61°7	0'67	2	4·8, 6·5	'678
Hussey 929 ...	18 23	76 33	112°0	1'54	1	8·6, 11·0	'701
Σ 2368 ...	18 36	52 16	325°5	1'91	2	7·2, 7·4	'837
Σ 2384 ...	18 38	67 2	314°3	0'82	2	8·0, 8·5	'630
Σ 2403 ...	18 43	60 57	264°4	1'42	2	6·2, 9·0	'659
Σ 2410 ...	18 45	59 14	94°7	1'47	3	8·2, 8·7	'663
Hussey 938 ...	18 54	64 25	218°3	1'82	2	9·1, 9·5	'671
Σ 2438 ...	18 56	58 6	30°4	0'65	2	7·0, 7·6	'676
Σ 2451 ...	19 0	51 28	69°5	2'26	1	8·7, 9·0	'841
Σ 2478 ...	19 2	69 19	299°8	1'15	2	8·8, 8·8	'648
Δ 19 AB	19 6	55 11	18°1	0'72	1	7·0, 8·0	'665
Σ 2479 AB & C			32°6	6'93	2	9·4	'735
Σ 2614 ...	19 7	88 12	235°1	1'53	2	8·8, 9·5	'738
OΣ 369 ...	19 8	71 56	28°9	0'96	2	7·0, 7·3	'634
Aitken 1174 ...	19 9	72 43	93°2	0'96	2	8·8, 9·5	'775
Hussey 943 ...	19 10	62 15	107°7	0'84	1	9·0, 10·0	'693
Σ 2509 ...	19 16	63 2	338°6	1'17	2	7·0, 8·1	'622
Espin 194 ...	19 20	64 21	217°7	5'46	2	8·8, 9·9	'690
Hussey 944 ...	19 26	66 46	355°0	1'54	2	9·0, 9·8	'841
Σ 2550 ...	19 29	73 10	69°1	2 02	2	8·2, 8·2	'682
Aitken 715 ...	19 32	59 50	343°1	0'59	1	9·1, 10·2	'690
Σ 2553 ...	19 32	61 51	101°7	0'85	2	8·4, 9·2	'703
Σ 2555 ...	19 34	53 10	283°0	1'87	1	8·5, 9·1	'805
Kustner 2 ...	19 35	71 24	269°2	1'03	2	7·2, 9·2	'699
β 656 ...	19 36	51 37	261°1	0'82	2	8·0, 9·2	'841
Aitken 864 ...	19 37	72 55	40°1	0'98	2	8·8, 9·0	'756
Espin 198 ...	19 38	64 43	313°3	3'39	2	8·8, 9·4	'658
Aitken 1191 ...	19 40	72 6	255°8	3'27	3	9·1, 9·1	'693
Σ 2603 ...	19 48	70 3	7°8	3'40	2	4·0, 7·6	'663
Σ 2605 ...	19 53	52 12	177°1	3'35	2	5·0, 7·5	'831
Σ 2623 ...	19 57	59 13	102°7	1'48	2	8·9, 10·9	'761
Hussey 956 ...	20 1	76 16	109°0	0'97	2	9·0, 10·0	'763
Aitken 865 ...	20 1	70 12	87°4	2'39	2	8·0, 10·4	'713
Σ 2640 ...	20 3	63 38	21°7	5'62	3	6·0, 9·9	'775
β 470 ...	20 4	63 30	214°6	3'05	2	9·5, 11·0	'696

Name of Star.	R.A. 1910.	Decl. 1910.	Position Angle.	Dist.	No. of Nights.	Mags.	Epoch 1909+.
	h m	° '	°	"			
Σ 2642 ...	... 20 4	63 26	175°7	2°40	3	8·7, 8·7	·699
Σ 2652 ...	... 20 7	61 48	261°6	0°41	1	7·3, 7·6	·832
Σ 2675 ...	... 20 12	77 26	120°0	7°79	1	4·0, 8·0	·857
Krueger 49	... 20 14	55 26	119°1	1°69	2	9·5, 9·7	·845
* ...	... 20 14	57 1	101°7	3°61	3	11·0, 12·0	·670
Krueger 50	... 20 15	57 1	131°3	2°45	2	9·0, 9·5	·629
Σ 2671 ...	... 20 16	55 7	339°6	3°38	2	6·0, 7·4	·745
Aitken 724	... 20 16	59 57	129°9	0°92	2	9·1, 9·8	·787
Hussey 958	... 20 18	62 19	343°7	0°77	1	9·0, 10·0	·832
Aitken 869	... 20 22	75 48	272°1	2 02	2	9·1, 9·5	·690
Aitken 729	... 20 22	57 49	251°1	2°82	1	9·0, 11·0	·805
OΣ 526 ...	... 20 22	80 53	171°2	1°10	2	7·8, 10·0	·657
Aitken 733	... 20 26	59 57	165°1	1°18	2	8·0, 10·0	·853
Hussey 761	... 20 29	60 48	117°3	0°61	3	8·8, 8·8	·725
Aitken 741	... 20 33	57 49	150°5	1°35	2	8·2, 11·5	·853
Σ 2717 ...	... 20 35	60 26	261°5	2°03	2	7·2, 9·7	·742
β 152 ...	... 20 40	57 3	107°8	0°56	2	7·2, 8·0	·797
β 471 ...	... 20 41	62 6	307°8	2°28	2	10·0, 10·0	·766
Krueger 51	... 20 42	58 42	179°1	1°48	2	9·0, 10·0	·828
Σ 2732 ...	... 20 45	51 35	72°6	4°13	2	6·7, 8·7	·738
Hussey 762	... 20 50	61 6	157°7	1°69	2	8·7, 10·0	·794
Aitken 752	... 20 52	56 30	14°0	1°21	2	9·0, 9·5	·853
Aitken 754	... 20 53	58 45	15°0	0°78	1	8·5, 9·5	·693
Σ 2740 ..	... 20 54	61 13	329°7	4°39	2	7·7, 10·0	·738
Aitken 756	... 20 55	58 28	238°5	0°63	1	7·3, 8·0	·816
Σ 3133 ...	... 20 55	61 1	102°1	3°67	2	7·4, 8·9	·705
β 472 ...	... 20 57	61 31	4°7	0°85	2	8·2, 8·5	·758
Σ 2751 ...	... 20 59	56 19	351°2	1°86	2	6·0, 7·0	·689
Hussey 959	... 21 0	66 21	156°8	1°49	3	7·7, 9·0	·746
Σ 2757 ...	... 21 1	52 2	267°5	1°87	2	7·8, 9·3	·845
Σ 2771 ...	... 21 3	70 24	212°9	2°96	2	8·8, 8·8	·697
Σ 2766 ...	... 21 4	58 38	249°7	4°95	2	8·3, 8·5	·705
Hussey 766	... 21 5	61 51	119°5	0°74	1	8·5, 10·0	·832
Aitken 760	... 21 5	58 45	330°1	3°57	2	9·0, 10·5	·827
Doolittle 16	... 21 8	55 58	32°7	0°97	2	8·6, 9·1	·849
Hussey 960	... 21 9	65 26	36°1	3°55	3	9·0, 11·3	·841
Σ 2780 ...	... 21 9	59 37	222°5	1°19	2	6·2, 7·2	·695
Σ 2783 ...	... 21 11	57 55	29°4	1°03	2	8·0, 8·0	·701
H I. 48 ...	... 21 12	64 2	269	...	1	7·0, 7·2	·928

Name of Star.	R.A. 1910. h m	Decl. 1910. ° ' "	Position Angle. °	Dist. "	No. of Nights.	Mags.	Epoch 1909+.
Σ 2790 ...	21 16	58 15	45°0	4'56	1	5'6, 9'9	'928
Σ 2795 ...	21 19	60 19	300°8	1'46	2	8'7, 9'5	'849
Σ 2801 ...	21 21	79 58	272°1	1'74	2	7'3, 8'0	'737
Σ 2807 ...	21 22	82 8	316°1	2'19	2	8'2, 8'3	'762
β 370 ...	21 29	52 21	326°7	3'65	2	8'5, 9'0	'841
Hussey 771 ...	21 29	77 32	193°9	2'53	1	7'0, 11'0	'690
OΣ 442 ...	21 31	61 24	358°1	0'75	1	8'0, 8'2	'693
β 687 ...	21 35	55 23	359°7	0'77	2	8'0, 9'0	'898
Hussey 968 ...	21 38	67 59	141°3	0'91	1	8'5, 10'0	'873
Σ 2837 ...	21 41	82 31	292°7	2'60	2	8'5, 9'0	'774
Σ 2827 ..	21 41	63 11	205°3	5'25	2	8'5, 9'0	'734
Holden 48 ...	21 45	51 11	21°7	4'62	2	8'6, 8'9	'853
Σ 2843 ...	21 49	65 19	138°0	1'98	3	7'0, 7'2	'723
OΣ 456 ...	21 52	52 6	28°5	1'55	2	7'8, 8'0	'833
OΣ 458 ...	21 53	59 22	349°6	0'80	2	7'1, 8'6	'931
Aitken 779 ...	21 56	60 7	270°4	0'49	1	7'9, 8'5	'934
Hussey 976 ...	21 58	62 24	40°0	1'57	2	9'0, '0	'779
Σ 2860 ...	22 0	60 25	257°2	7'76	2	7'7, 9'3	'775
Σ 2863 ...	22 1	64 11	281°5	7'00	2	4'7, 6'5	'729
Espin 103 ...	22 2	53 57	212°3	1'79	2	9'1, 9'3	'837
Σ 2872 A & BC ...	22 5	58 51	316°3	21'73	1	7'2	'857
„ B & C			317°0	0'66	2	8'0, 8'0	'931
Σ 2879 ...	22 8	62 57	230°5	0'77	2	8'0, 8'0	'872
Aitken 626 ...	22 9	59 46	259°9	0'55	1	9'0, 9'0	'928
β 991 ...	22 10	52 7	144°6	0'57	1	8'0, 8'0	'928
Hough 614 ...	22 11	51 2	174°9	4'86	2	7'5, 10'0	'859
Krueger 57 ...	22 13	61 35	226°1	1'66	3	9'0, 9'1	'824
Krueger 58 ...	22 17	59 25	31°7	2'04	2	9'0, 9'1	'797
Aitken 629 ...	22 19	59 32	331°8	1'02	2	9'2, 9'4	'931
Krueger 59 ...	22 23	63 15	164°6	1'86	2	9'0, 9'2	'826
Krueger 60 A & B	22 24	57 15	94°1	3'02	1	9'0, 12'0	'928
„ AC			59°0	45'26	1	9'0, 9'2	'928
Σ 2924 ...	22 30	69 26	278°3	0'80	2	6'8, 7'3	'803
Hussey 984 ...	22 43	65 47	18°3	0'66	2	10 10	'878
Σ 2947 ...	22 45	68 5	64°7	3'97	3	7'2, 7'2	'753
OΣ 530 ...	22 46	67 42	209°9	5'55	2	9'0, 10'0	'796
Holden 54 ...	22 46	50 39	13°5	1'72	2	8'7, 8'9	'836
β 1332 ...	22 47	52 34	129°8	1'75	2	8'4, 8'6	'859
Σ 2950 ...	22 47	61 13	306°5	2'12	3	5'7, 7'0	'777

Name of Star.	R.A. 1910.	Decl. 1910.	Position Angle.	Dist.	No. of Nights.	Mags.	Epoch 1909+.
	h m	° '	°	"			
Aitken 632 ...	22 48	57 15	115°6	0'47	2	8'0, 8'8	'931
Hussey 993 ...	22 58	67 18	219°1	1'76	2	7'9, 10'2	'859
* ...	23 5	50 58	151°8	7'06	2	9'5, 9'8	'859
Holden 57 ...	23 5	51 3	294°4	2'26	2	8'7, 10'3	'859
OΣ 490 ...	23 6	56 58	303°3	1'32	1	7'2, 9'2	'852
Hussey 996 A & B	23 8	67 6	209°0	2'44	2	8'5, 9'1	'828
„ A & C			184°9	18'61	1	11 12	'873
Σ 2996 ...	23 9	81 20	107°3	5'04	3	8'3, 8'7	'789
Aitken 787 ...	23 10	68 36	195°1	2'82	1	9'3, 9'4	'821
β 853 ...	23 12	61 19	230°4	0'70	1	8'7, 8'7	'852
Σ 3001 ...	23 14	67 37	199°0	3'09	2	5'2, 7'8	'773
Aitken 640 ...	23 17	60 5	14°7	0'88	1	9'4, 9'6	'852
Espin 108 ...	23 21	52 9	247°0	3'07	2	9'1, 9'2	'892
β 1150 ...	23 26	64 34	56°4	0'63	1	8'7, 9'0	'852
β 774 ...	23 26	63 50	2°0	0'65	1	8'4, 8'8	'852
β 855 ...	23 33	67 43	202°7	0'87	2	8'5, 8'8	'953
β 856 ...	23 34	70 8	271°4	0'57	1	8'1, 9'1	'852
β 857 ...	23 36	67 3	300°1	1'54	2	8'5, 8'9	'802
β 1152 ...	23 43	63 19	108°4	0'60	2	9'2, 9'2	'950
Hussey 797 ...	23 44	82 17	133°5	1'10	2	8'8, 9'0	'900
OΣ 507 AB	23 44	64 23	268°8	0'69	2	6'8, 7'5	'890
AB & C ...			352°1	49'27	2	7'8	'890
Σ 3047 A & B {	23 53	56 53	70°3	1'05	2	8'7, 8'7	'880
β 280 A & C }			192°1	8'93	1	8'7, 12'5	'934
Σ 3049 ...	23 54	55 15	327°0	3'20	3	5'4, 7'5	'790
β 1154 ...	23 54	74 20	313°7	0'96	2	8'0, 8'2	'916
Holden 59 AB	23 55	52 45	12°7	1'15	1	8'6, 8'8	'980
AB & C			308°4	18'55	1	10'8	'980
β 861 ...	23 58	69 12	179°6	1'90	2	9'4, 9'7	'953

Edinburgh, 1910 March.

Southern Double Star Measures. 4¼-in. Cooke Refractor.

By G. D. Hirst.

Each of these measures is the mean of a large number of observations made on one night only, the night selected being one in which the seeing was considered as well-nigh perfect.

Star.	Epoch.	R.A.	Dec. South.	Mag.	Angle.	Dist.	Remarks.
		h m s	° ' "		°	"	
β Toucani	1906.8	0 26 58	63 31	4.3 4.5	350°0	25.1	
β Phœnicis	1906.8	1 1 37	47 15	4.2 4.2	16°5	1.2±	Distance not altered appreciably. Angle decreasing.
ζ Phœnicis	1907.9	1 4 11	55 46	4.1 8.4	242°2	6.1	No apparent change.
κ Toucani	1906.9	1 12 23	69 42	5.0 7.7	350°9	5.17	Yellow-greenish.
τ Sculptoris	1906.9	1 31 31	30 25	6.1 7.1	95°0	2.0	Distance and angle appear unaltered.
ρ Eridani	1906.7	1 35 59	56 42	6.0 6.1	221°1	8.6	Measures from following stars.
"	1908.9	"	"	"	217°4	7.7	Good measures—distance diminishing.
θ Eridani	1906.9	2 54 28	40 42	3.3 4.8	86°0	8.7	Measured in daylight.
α Fornacis	1906.9	3 7 49	29 22	3.9 6.9			Absolutely round with all powers.
h 19	1906.9	3 8 55	44 47	A. 6.5 B. 7.0 C. 10.1		A. B. 0.9±	A.B. discs in contact—most difficult object, C. 10.8 mag.
f Eridani	1908.1	3 44 54	37 55	4.9 5.6	206°5	7.5	Apparently fixed.
θ Reticuli	1907.0	4 16 33	63 29	6.3 8.4	6°5	3.6	Previous measures very discordant. Stars steady—good obs.
Lac. 1475	1908.9	4 42 15	57 17	7.0 7.7	236°5	6.3	
δ Argus	1908.2	8 41 56	54 20	2.1 5.2	169°8	2.9	Previous measures discordant. Small star 6.5 mag.
Lac. 3917	1908.2	9 30 9	48 33	5.8 6.4	210°0	3.7	An interesting double—possibly binary. Angle and distance increasing.
U Argus	1908.2	9 44 36	64 36	3.0 7.0	125°9	5.0	No change since early measures.
Lac. 4297	1908.2	10 10 21	83 35	7.8 8.3	271°1	4.8	Small decrease in angle, increase in distance.

Mar. 1910.

Southern Double Star Measures.

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Star.	Epoch.	R.A.			Dec. South.	Mag.	Angle.	Dist.	Remarks.
		h	m	s	°		°	"	
usæ	1907.4	12	40	9	67 33	3.9 4.2	345.5	1.10	Angle increasing, older measures somewhat discordant—apparently no change in distance since 1900.
ntauri	1909.5	13	35	0	54 3	5.8 7.1	165.7	5.01	
ntauri	1909.5	13	46	0	32 29	4.7 5.9	107.3	7.3	
5750	1909.5	13	50	40	55 32	7.2 8.6			Small star too faint to measure, 9 mag.
5893	1909.5	14	15	27	58	5.2 6.8	160.6	8.8	
ntauri	1906.6	14	32	38	60 25	0.4	212.5	21.4	Measured in daylight.
	1908.6					1.9	214.6	19.9	"
	1909.6						214.7	19.32	"
6477	1909.5	15	38	46	65 7	6.5 6.6	151.0	1.46	Some slight change.
pi	1909.5	15	53	30	38 6	3.8 7.8	21.2	14.5	Small star 8 m., dusky yellow.
ronæ Aust.	1906.7	18	59	40	37 12	5.1 5.1	117.8	2.0	
etantis	1908.8	20	29	49	75 41	6.9 7.9	16.9	16.8	
di	1906.7	21	12	44	53 52	4.7 7.2	284.5	5.9	Distance increasing.
uis	1906.7	23	1	15	44 3	4.5 8.2	30.0	± 1.4	Angle increasing—dist. diminishing.
9367	1909.8	23	1	28	51 13	6.6 7.1	257.6	8.0	Small change in angle.
9446	1909.8	23	15	13	50 51	6.3 7.5	209.9	16.0	Small star 8 mag., dusky—no change.

Sydney, 1910 January 30.

A New Nebula. By the Rev. T. E. Espin, M.A.

On the night of January 12 a new nebula was detected, which has hitherto escaped notice. The following are the detailed observations of it:—

Jan. 12. Small, fairly bright, with star-like nucleus near a star, Mag. 11.5.

Jan. 19. Star and nucleus measured.

Feb. 10 Well seen between clouds, elongated P $320^{\circ} \pm$, pretty bright, $15''$ by $10''$ estimated. Clouds prevented measures.

Feb. 11. Pretty bright, and seems about $10''$.

March 5. Bright, and apparently elongated N.p. nucleus equal 13 mag.

The following micrometrical measures of the star and nucleus have been made:—

	1910.030	P 68.8	D 21.05	Mag. 11.5	13.0	
	.049	71.1	20.64			
	.112	71.1	21.40	12.0	13.0	
Means	1910.064	70.3	21.03	11.7	13.0	3nts.

It is somewhat remarkable that this object has hitherto escaped detection, as there is a cluster of nebulae, thirteen in all, the centre of the group being some $20'$ N and 1^m p. the new nebula. It is smaller, but much brighter than most of these; indeed, several of them were barely suspected on a dark sky with the $17\frac{1}{4}$ -inch, whereas the new nebula could be seen in full moonlight.

The place of the nebula was determined from B.D. + 50° 1399 and B.D. + 50° 1401, on three nights the mean result being:—

R.A. $7^h 4^m 28.85$ Decl. + $50^{\circ} 1' 33''.0$ 1910.0.

With a low-power eyepiece the position angles were measured from these two stars as follows:—

B.D. + 50° 1399 and nebula P $109^{\circ}.4$ 3nts.

B.D. + 50° 1401 „ „ P $330^{\circ}.4$ 3nts.

Corrections to Colour-spectrum Discordances.

By W. S. Franks.

Through the courtesy of Professor Pickering, I have been favoured with the result of a re-examination by Mrs. Fleming of the spectra quoted on p. 191, vol. lxx., of the *Monthly Notices*, in my paper on "Star Colours and Spectral Types."

H.P.	Spectrum.	Re-exam.	Remarks.
589	B5	B8?	But B5 in <i>H.A.</i> , vol. lx. p. 73
7763	B4	Pec.	Spectrum peculiar, with bright lines.
7391	A	K	Wrongly identified in <i>H.A.</i> , l. 236; <i>D.C.</i> correct.
7759	A	Kp	} Spectrum wrong in <i>H.A.</i> , l.
8306	A	K	
8857	A	G5	
2394	F	K	
2241	F5	F2	Spectrum F5 in <i>H.A.</i> , lx. 73.
3779	F5	K	
7904	F5	G	
996	K	G	Spectrum G5 in <i>H.A.</i> , lx. 75; type III.?
5853	K	G	
6638	K	K	
6849	K	F	Spectrum wrong in <i>H.A.</i> , l.
8729	K	F8	
5940	K5	Ma	Type III.
4504	Ma	K	
4964	Ma	Ma	

There is thus a substantial decrease in the number of discordances, with a corresponding increase in the value of the coefficient of mean square contingency. Some of the remaining discrepancies are evidently due to real differences between the photographic and visual spectra.

Uxbridge:

1910 February 24.

*On the Theory of Aberration and the Principle of Relativity.
An Explanation.* By H. C. Plummer, M.A.

In the course of my paper under the above title, reference was made to Villarceau's contribution to the subject of secular aberration. Inasmuch as Professor Seeliger had obtained results which were not in accordance with Villarceau's conclusion, and he had been followed by a recent writer on the same subject, it was felt necessary to refer (p. 260) to his paper in *A.N.*, No. 2610 (1884), and to criticise his theory.*

It should, however, be pointed out that in *A.N.*, No. 4263 (1908), Professor Seeliger has published another paper in which he has himself arrived at the same conclusion as Villarceau. With this paper I was unfortunately unacquainted, and I wish to express my regret that my criticism appeared without any allusion to Professor Seeliger's own later view.

On Mr. Plummer's Operator K. By T. C. Hudson.

On reading Mr. Plummer's paper† on Interpolation, it occurred to me that his K is $4 \sinh^2 \frac{1}{2}wD$, $K^\dagger$ having thus a natural interpretation as $2 \sinh \frac{1}{2}wD$. The other operator k is, I think, best regarded as complex and made up of $2 \sinh \frac{1}{2}wD$ and $\cosh \frac{1}{2}wD$. Of these two, the first is the pure operator for finite differences, and the second for finite means. They both operate, not on the upper nor on the lower tabulated function, but on the function midway. In this connection I may point out that the Δ and the δ of Everett's diagram are supposed to operate on functions not on their level; consequently they are complicated by entanglement with $\exp. \pm \frac{1}{2}wD$ the operators of semi-enlargement.

Change of sign of D reverses the operator of enlargement, the operation, and the order of the series of functions. Taking means and half-differences of the original and the reversed series of operators we get a pair of series of new operators typified by $\cosh \frac{1}{2}wD$ and $\sinh \frac{1}{2}wD$. These two, and also their respective products into $\text{sech} \frac{1}{2}wD$, may be expanded in terms of powers of $\sinh \frac{1}{2}wD$, giving the usual formula for interpolation with central differences.

In like manner, expansion of $\sinh \frac{1}{2}wD$, and its powers, in terms of $\frac{1}{2}wD$, and *vice versa*, gives us relations between differences and differential coefficients, and so forth.

For these and similar reasons I suggest that $K^\dagger = 2 \sinh \frac{1}{2}wD$

* For an earlier criticism, cf. *Bull. Astr.*, iv. p. 94 (1887). For this reference I am indebted to the kindness of Professor Seeliger himself.

† *M.N.*, lxx. 3, p. 266.

should be considered as the operator for finite differences, and $2 \cosh \frac{1}{2}wD$ for finite sums.

Since arriving at these results, I find that Mr. Sheppard, in papers* published simultaneously with Everett's, has stated views substantially identical with those expressed here.

H.M. Naut. Alm. Office.

Note on the above paper by H. C. Plummer, M.A.

The expressions for the operators K and k in terms of D by means of hyperbolic functions were known to me at the time when my paper was written, and they were deliberately avoided. As, however, they form the connection with other work on the subject, in particular with Mr. Sheppard's paper on Central Difference Formulæ (with which I was familiar), Mr. Hudson is perfectly right in calling attention to these relations.

My reason for avoiding the use of these expressions was simple. The results which were required are, in point of fact, in the form of power-series in K , associated with another operator which occurs, if at all, only in a linear form. Hence there is an obvious advantage in working with these operators explicitly from the beginning. It is found that the required formulæ can in this way be developed very succinctly from the first principles of algebra. From this point of view the composite character of the operators is beside the point. It seems to me the positive advantage of the method adopted that it enables us to ignore this composite character.

On the other hand, it is perfectly possible to employ the expressions in terms of hyperbolic functions, and to assume the known expansions of these functions and of certain forms in which they occur. This is the method of Mr. Sheppard, who has developed the theory very completely on those lines. Mr. Hudson very justly emphasises the value of this procedure. But my own object was to avoid the necessity of quoting any expansions of the kind, and hence it was foreign to the immediate purpose of my paper to introduce the hyperbolic functions at all.

The following reports of Observatories were received too late for insertion in the Annual Report of the Council:—

Sydney Observatory.

(*Mr. W. E. Raymond, Officer in Charge.*)

The weather during the year 1909 was, on the whole, more favourable for observing than that of the previous year, and a good number of observations have been made. The Transit Meridian

* *Proc. Lond. Math. Soc.*, xxxi. pp. 453, 462.

Mr. Tebbutt's Observatory, Windsor, New South Wales.

For reasons previously stated, very little has been done during the year 1909. The astronomical work consists of measures of well-known binary stars, and a few determinations of position of Halley's Comet by means of the 8-inch equatorial. The usual meteorological observations have been continued.

Errata in Annual Report, vol. lxx.

Page 302, lines 23 and 25, *for* Mitchell *read* Mitchel.

Page 358, line 5 from bottom, *for* U Herculis *read* u Herculis.

